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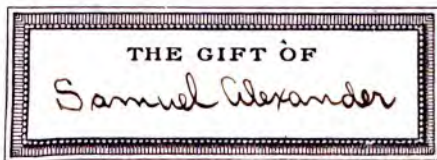
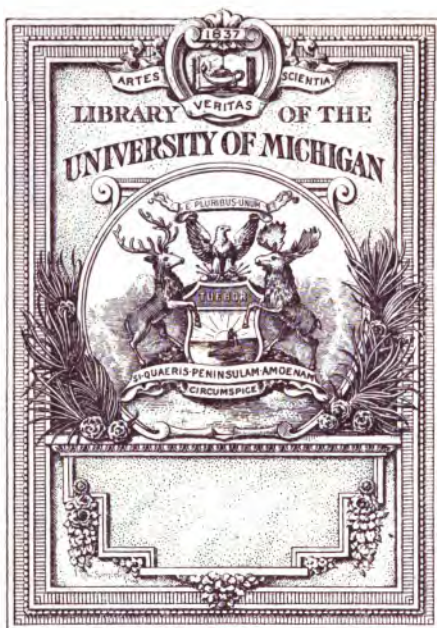
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to in the Preface of this Book. *The Eighth
Edition.*



THE PREFACE.



HAVING perused several Books concerning the Mensuration of Surfaces and Solids, and the Works of Artificers relating to Building; but not finding any one Book so perfect, as to give any tolerable Satisfaction to a Learner; and I having practised and taught Measuring for several Years, and thereby gain'd Experience and Knowledge in that Art, having learned some Things from one Author, and some Things from another, I began to think of digesting my Thoughts into some such Method as might give a Learner full Satisfaction, without being at the Charge of buying so many Books; and being importun'd thereunto by some Friends, I fell to work, and at last brought them to that Perfection you here find in the following Work.

1. As to the *Decimal Arithmetic*, I have been as brief as the Matter would well bear, to make it plain.

A 2

2. A

156874

2. As to the Multiplying of Feet and Inches, commonly call'd *Cross-Multiplication*; my Method differs from that which is usually taught in other Authors, as being (I think) much shorter and plainer.

3. In measuring of Superficies and Solids, I have given the Demonstration of the Rules, which I thought might be very acceptable to the Ingenious; for, indeed, I always look upon the Writing of a Rule without a Demonstration (in any Part of the Mathematicks) to be but lame and defective; and for want of knowing the Reason of the Rule, a Learner may commit great Errors; besides, when a Learner knows the Reason of the Rules, he may retain them better in his Memory. The Rule for measuring a Prismoid and Cylindroid, I had out of Mr. *Everard's Art of Gauging*; but the Reason he does not shew, neither have I found it in any other Author; but that the Method is true, I have endeavour'd to make plain.

The Demonstration of the Rules for finding the Area of an Ellipsis and Parabola; also the Demonstration of the Rules for finding the solid Content of the Frustum of a Cone and Pyramid, the Solidity of a Globe of a Spheroid, a Parabolic Conoid, and of a Parabolic Spindle, and their Frustums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide*; where the Curious and ingenious Reader may see many other Demonstrations algebraically perform'd. I have also demonstrated the Rule for finding the Solidity of a Globe out of *Pardic's Elements of Geometry* (Book the 5th, Art. the 33d) publish'd in *English* with many.

The PREFACE.

v

ny Additions, by the Reverend Dr. *Harris*, F. R. S. and the same is also done out of *Sturmius's Mathe-
fis Eucleata*; so that the ingenious Reader may use
which of those Ways he likes best.

The Scale suppos'd to be used in all the Opera-
tions, is the Line of Numbers, commonly call'd,
Gunter's Line, which is upon the ordinary Two-
Feet, or Eighteen-Inch Rules, commonly used by
the Carpenters, Masons, &c. because I thought
it needless, as well as impertinent, to write the Use
of Sliding-rules, or any other particular Scales, they
being sufficiently treated of by several Authors, viz.
by the above-nam'd Mr. *Everard*, in his *Art of
Gauging* above-mentioned, where you have the Use
of a Sliding-rule in Arithmetic, Geometry, in
Measuring of Superficies and Solids, Gauging, &c.
Likewise Mr. *Hunt* has written largely of the Uses of
his Sliding-rule, in Arithmetic, Geometry, Trigo-
nometry, Gauging, Dialing, &c. There are several
others who have explain'd the Use of their
own Rules; so that the more curious Readers may
find full Satisfaction in those Authors.

One thing I have omitted in the Book, which
I think may not be very improperly inserted in this
Place; that is, how to find a Number upon the
Line. If the Number you would find consists
only of Units, then the Figures upon the Line repre-
sent the Number sought: Thus, if the Number be
1, 2, 3, &c. then 1, 2, 3, &c. upon the Line, re-
presents the Number sought. But if the Number con-
sists of two Figures, that is, of Units and Tens, then
the Figure upon the Rule stands for the Tens, and

the large Divisions stand for the Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Divisions (counted forwards) is the Point representing 34; and if 340 were to be found, it will be at the same Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and four of the smaller Divisions (counted forward) is the Point representing 304. If the Number consists of four Places, or Thousands, then the Figure upon the Line stands for Thousands, and the larger Divisions are Hundreds, the lesser Divisions are Tens, and the tenth Parts of those lesser Divisions are Units. Thus, if 2735 were to be found, then the 2 is 2000; and the 7 larger Divisions (counted forward) is 700 more; and 3 of the lesser Divisions is 30 more; and half of one of the lesser Divisions is 5 more, which is the Point representing 2735. You must remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Divisions; and each of those larger Divisions are subdivided (or supposed so to be) into 10 other Parts, which I call the smaller Divisions; and each of those Parts supposed to be subdivided again into 10 other Parts, &c. You must also remember, that if 1, in the Middle of the Line, stands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End signifies 1, then 1 in the Middle stands for 10, and 1 at the upper End is 100, &c.

There is one Thing more which I would have my Reader to understand, and that is, How to find all such proportional Numbers made use of in the Proportions about a Circle, and of a Cylinder, and in

The PREFACE. vii

in other Places; which Thing may be of good Use; to know how to correct a Number, which may happen to be false printed, or to enlarge any Number to more decimal Places, for more Exactness; for tho' I have mention'd what such Numbers are, yet I have not shewn how to find them, which a Learner may be a little at a Nonplus to do; tho' they are easily found by the Rules there laid down. I shall therefore give two or three Examples, in this Place, of finding such Numbers, which may enable my Reader to find out the rest.

And, first, let it be requir'd to find the Area of a Circle, whose Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, &c. whereof 3.1416 is sufficient in most Cases. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by .5 (*viz.* half 3.1416 by half 1) and the Product is .7854 which is the Area of the Circle, whose Diameter is 1.

Again, If the Area be required when the Circumference is 1, first, find what the Diameter will be, thus: As 3.1416 : to 1 :: so is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is .159154 by .5, and the Product is .079577, which is the Area of a Circle whose Circumference is 1.

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7853 is .8862, which is the Side of a
Square

Square equal when the Diameter is 1. And if you extract the square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whose Circumference is 1.

If the Side of a Square within a Circle be requir'd, if you square the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that shall be the Side of the Square which may be inscrib'd in that Circle; so, if the Diameter of the Circle be 1, then the Half is .5; which, squar'd, is .25; and this, doubled, is .5, whose Square Root is .7071, the Side of the Square inscrib'd.

Again, If the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonstrated, that the Globe is $\frac{8}{3}$ of a Cylinder of the same Diameter and Altitude: Thus if the Cylinder's Diameter be 1, and its Altitude or Length be also 1, find the Solidity thereof, and take $\frac{8}{3}$ of it, and that will be the Solidity of the Globe requir'd. Now if the Diameter be 1, the Area of the Circle, or Base of the Cylinder, is .7854, (as is above shewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is also .7854, the Solidity of the Cylinder; $\frac{8}{3}$ whereof is .5236, which is the Solidity of the Globe, whose Diameter is 1.

From what has been said, the Reader may easily perceive how all other proportional Numbers are found, and may examine them at his Pleasure.

I shall not enlarge any further upon the Matter, but leave the Book to speak for itself; and if it prove beneficial to the ingenious Practitioners,

The **P R E F A C E.** ix

ners, I have my Desire. So, wishing my ingenious Reader good Success in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Desire of his Well-wisher,

W. HAWNEY,





THE CONTENTS.

PART I.

Chap.	Page
I. <i>What a Decimal Fraction is</i>	1
II. <i>Reduction of Decimals</i>	3
III. <i>Addition of Decimals</i>	9
IV. <i>Subtraction of Decimals</i>	10
V. <i>Multiplication of Decimals</i>	11
Ibid. <i>Contracted Multiplication</i>	14
VI. <i>Division of Decimals</i>	19
Ibid. <i>Contracted Division</i>	26
VII. <i>Extraction of the Square Root</i>	33
VIII. <i>Extraction of the Cube Root</i>	43
IX. <i>Cross Multiplication</i>	58

PART II. CHAP. I.

Seçt.	Page
1. <i>Of a Square</i>	71
2. <i>Of a Parallelogram</i>	73
3. <i>Of a Rhombus</i>	74
4. <i>Of a Rhomboides</i>	75
5. <i>Of</i>	

The CONTENTS.

Sect.	Page
5. Of a Triangle	76
6. Of a Trapezium	82
7. Of an irregular Figure	84
8. Of regular Polygons	86
9. Of a Circle	90
10. Of a Semi-Circle	110
11. Of a Quadrant	111
Ibid. To find the Length of the Arch Line	112
Ibid. By having the Chord and versed Sine, to find the Diameter	115
12. Of a Sector of a Circle	116
13. Of the Segment of a Circle	118
14. Of compound Figures	122
15. Of an Ellipsis, or Oval	124
16. Of a Parabola	128

CHAP. II.

Of Solid Measure.

Sect.	Page
1. Of a Cube	134
2. Of a Parallelopipedon	137
3. Of a Prism	140
4. Of a Pyramid	145
5. Of a Cylinder	154
6. Of a Cone	156
7. Of the Frustum of a Pyramid	160
8. Of the Frustum of a Cone	172
9. Of a Prismoid	176
10. Of a Cyliindroid	180
11. Of a Sphere, or Globe	182
12. Of a Spheroid	198
13. Of a Parabolic Conoid	198
14. Of a Parabolic Spindle	201

CHAP.

The CONTENTS.

CHAP. III.

The Measuring of Works relating to Building.

Sect.	Page
1. Of Carpenters Work	206
2. Of Bricklayers Work	211
3. Of Plasterers Work	223
4. Of Joiners Work	225
5. Of Painters Work	228
6. Of Glasiers Work	229
7. Of Masons Work	232

CHAP. IV.

Sect.	Page
1. Of Board Measure	235
2. Of squar'd Timber	237
3. Of unequal squar'd Timber	245
4. Of round Timber	249
5. Of round Timber with unequal Bases	258
6. Of the Five regular Bodies.	263
7. Of irregular Solids	271

CHAP V.

<i>Practical Questions</i>	273
----------------------------	-----

A P P E N D I X.

Sect.	Page
1. Of Gauging	309
Of Land-measuring	335

THE



THE Complete MEASURER.

PART I.

CHAP. I.

Notation of DECIMALS.



DECIMAL Fraction is an artificial Way of setting down and expressing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annex'd to it, and must therefore be either 10, 100, 1000, 10000, &c. and therefore, in writing down of a Decimal Fraction, there is no Necessity of writing down the Denominator; for by bare Inspection it is certainly known, it consisting of a Unit, with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

B

Example

2 *Notation of DECIMALS.* Part I.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus .25, its Denominator being known to be an Unit with two Cyphers; because there are two Figures in the Numerator. In like manner, $\frac{3575}{10000}$ may be thus written, .125; and $\frac{3575}{100000}$ thus, .3575; and $\frac{75}{10000}$ thus, .075; and $\frac{65}{100000}$ thus, .0065.

As whole Numbers increase in a decuple or tenfold Proportion, towards the Left Hand, so, on the contrary, Decimals decrease towards the Right Hand in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thousandth Parts.	Ten Thousandth Parts.	Hundred Thousandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right Hand of whole Numbers, do increase the Value of those Numbers in a decuple (or tenfold) Proportion; but being annexed to the Right Hand of a Decimal Fraction, do neither increase nor decrease the Value thereof: So $\frac{25}{100000}$ is equivalent to $\frac{25}{1000}$ or .25. And, on the contrary, tho' in whole Numbers, Cyphers prefix'd before them, do neither increase nor diminish the Value; yet Cyphers before a decimal Fraction do diminish its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{25}{10000}$ or .025: and .125 is $\frac{125}{100000}$, by prefixing two Cypher before it, thus, .00125. And therefore, when you are to write a Decimal Fraction, whose Denominator hath more Cyphers than there are Figures in

Chap. 2. *Reduction of DECIMALS.* 3

the Numerator, they must be supply'd by prefixing so many Cyphers before the Figures of your Numerator; as, suppose $\frac{19}{1000}$ were to be written down, without its Denominator; here, because there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and set it down thus, .019.

The Integers are separated from the Decimals several ways, according to Mens Fancies; but the best and most usual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is sufficient: Thus, if you were to write down 317 $\frac{217}{1000}$ it may be thus express'd, 317.217; and 59 $\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, &c.



CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Cases: 1st, To reduce a vulgar Fraction to a Decimal. 2^{dly}, To find the Value of a Decimal in the known Parts of Coin, Weights, Measures, &c. 3^{dly}, To reduce Coin, Weights, Measures, &c. to a Decimal. Of these in their Order.

I. To reduce a vulgar Fraction to a Decimal.

The RULE.

As the Denominator of the given Fraction is to its Numerator, so is an Unit (with a competent Number of Cyphers annex'd) to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Re-

4 *Reduction of DECIMALS.* Part I.

subt by the Denominator, the Quotient is the Decimal equivalent to the vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduc'd to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal requir'd, and is equivalent to $\frac{3}{4}$ given.

Note, That so many Cyphers as you annex to the given Numerator, so many Places must be prick'd off in the Decimal found; and if it shall happen, that there are not so many Places of Figures in the Quotient, the Deficiency must be supply'd, by prefixing so many Cyphers before the Quotient Figures, as in the next Example.

Example 2. Let $\frac{3}{3}$ be reduced to a Decimal having six Places.

To the Numerator annex six Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was requir'd to have six Places, therefore you must prefix two Cyphers before it, and then it will be 005235, which is the Decimal required, and is equivalent to $\frac{3}{3}$.

See the Work of these two Examples.

$$4 \overline{) 3.00(.75}$$

28

20

20

..

$$573 \overline{) 3.000000(.005235}$$

....

1350

2040

3210

345

In the second Example there remains 345, which Remainder is very insignificant, it being less than $\frac{1}{100000}$ Part of an Unit, and therefore is rejected.

II. To

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Measures, &c.*

The R U L E.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off so many Places to the Right Hand as there were Places in the Decimal given; and multiply those Figures prick'd off by the Number of Parts in the next inferior Denomination, and prick off so many Places as before, and so continue to do, till you have brought it to the lowest Denomination requir'd.

Example 1. Let .7565 of a Pound Sterling be given to be reduc'd to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and by 4, as the Rule directs, and always prick off four Places to the Right Hand, and you will find it make 15 s. 1 d. 2 q. See the Work.

$$\begin{array}{r}
 .7565 \\
 \times 20 \\
 \hline
 15.1300 \\
 \times 12 \\
 \hline
 1.5600 \\
 \times 4 \\
 \hline
 2.2400
 \end{array}$$

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the first Figure, (or Place of Primes) and it makes so many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then

B 3 for

6 Reduction of DECIMALS. Part I.

for every Unit in the second Place count ten, and to that add the Figure in the third Place, and reckon that so many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

First, Double your 6, and it makes 12 s. then take 5 out of 9, and for that reckon another Shilling, and it makes 13 s. and the four remaining is 4 Tens, and the 5 makes 45, which being above 38, you must therefore cast away 2, and there rests 43 Farthings, which is 10 d. $\frac{3}{4}$. So the Answer is 13 s. 10 d. $\frac{3}{4}$.

	l.	s.	d.
So the Value of .725 =	14	6	
And the Value of .878 =	17	6	$\frac{1}{4}$
And the Value of .417 =	8	4	

And so of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right hand, and you will find the Answer to be 7 oz. 3 pwt. 13 gr. *ferè*. See the Work.

.59755	
12	
7.17060	
20	
3.41200	<i>oz. pwt. gr.</i>
24	<i>Facit 7 3 9.888</i>
164800	
82400	
9.88800	

Chap. 2. *Reduction of DECIMALS.* 7

Let .43569 of a Ton be reduc'd to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Answer will be 8 C. 2 qrs. 24 lb. fers.

$$\begin{array}{r}
 .43569 \\
 \text{20} \\
 \hline
 8.71380 \\
 \text{4} \quad \text{C. qrs. lb.} \\
 \hline
 \text{Facit 8} \quad 2 \quad 23.9456 \\
 2.85520 \\
 \text{28} \\
 \hline
 23.94560
 \end{array}$$

Let .9595 of a Foot be reduc'd into Inches and Quarters.

$$\begin{array}{r}
 .9595 \\
 \text{12} \\
 \hline
 11.5140 \\
 \text{4} \quad \text{Facit 11 Inches, 2 Quarters.} \\
 \hline
 2.0560
 \end{array}$$

III. *To reduce the known Parts of Money, Weight, Measure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the lesser Denomination given, annex a competent Number of Cyphers, and divide by the Number of such Parts that are contain'd in the greater Denomination, to which the Decimal is to be brought ; and the Quotient is the Decimal sought.

Example

8 *Reduction of DECIMALS.* Part I.

Example 1. Let 6d. be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers, (suppose 3) and divide the Result by 240, (the Pence in a Pound) and the Quotient is the Decimal requir'd.

$$\begin{array}{r} 240)6.000(.025 \\ \underline{1200} \\ 0000 \\ \underline{0000} \\ 0000 \\ \underline{0000} \\ 0000 \\ 0000 \end{array} \quad \text{Facit .025}$$

Example 2. Let $3d. \frac{3}{4}$ be reduc'd to the Decimal of a Pound, having six Places.

In 3 *d.* $\frac{2}{3}$ there are fifteen Farthings, therefore to 15 annex six Cyphers, (because there are to be six Places in the Decimal requir'd) and divide by 960, (the Farthings in a Pound) and the Quotient is .015625.

$$\begin{array}{r} 96'0''15.00000'0(.015625 \\ \text{.....} \\ \hline 540 \\ 600 \\ 240 \\ 480 \\ \hline \end{array}$$

Example 3. Let $3\frac{1}{3}$ Inches be reduc'd to the Decimal of a Foot, consisting of four Places.

In $3\frac{1}{4}$ Inches, there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48, (the Quarters in a Foot) and the Quotient is .2708.

$$\begin{array}{r} 48 \overline{) 13.0000(.2708} \\ \underline{340} \\ 400 \\ \underline{360} \\ 40 \end{array}$$

Example

Chap. 3. *Addition of DECIMALS.* 9

Example 4. Let 9 C 1 gr. 16 lb. be reduc'd to the Decimal of a Ton, having fix Places.

C. gr. lb.	
9 1 16	2240)1052.000000(.469642
4
37 grs-	15600
28	21600
	14400
302 <i>Facit</i> .469642	9600
75	6400
1052 Pounds	1920

~~~~~

## CHAP. V.

### *Addition of DECIMALS.*

**A**DDITION of Decimals is perform'd the same way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, primes under Primes, Seconds under Seconds, &c.

*Example.* Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

|                    |
|--------------------|
| 317.25             |
| 17.125             |
| 275.5              |
| 47.3579            |
| 12.75              |
| <hr/> Sum 669.9829 |

This is so plain, that more Examples I think needless.

CHAP.

## C H A P. IV.

*Subtraction of DECIMALS.*

**S**UBTRACTION of Decimals is perform'd likewise the same way as in whole Numbers, respect being had to the right placing the Numbers, (as in Addition) as in the following Examples.

( 1 )

|        |                 |
|--------|-----------------|
| From   | 212.0137        |
| Subtr. | <u>31.1275</u>  |
| Refts  | <u>180.8862</u> |
| Proof  | <u>212.0137</u> |

( 2 )

|        |                 |
|--------|-----------------|
| From   | 201.1250        |
| Subtr. | <u>5.5785</u>   |
| Refts  | <u>195.5465</u> |
| Proof  | <u>201.1250</u> |

( 3 )

|        |                 |
|--------|-----------------|
| From   | 2051.315        |
| Subtr. | <u>79.172</u>   |
| Refts  | <u>1972.143</u> |
| Proof  | <u>2051.315</u> |

( 4 )

|        |                |
|--------|----------------|
| From   | 30.5           |
| Subtr. | <u>7.2597</u>  |
| Refts  | <u>23.2403</u> |
| Proof  | <u>30.5</u>    |

*Note,* If the Number of Places in the Decimals be more in that which is to be subtracted, than in that which you subtract from, you must suppose Cyphers to make up the Number of Places, as in the fourth Example.

C H A P. V.

*Multiplication of DECIMALS.*

**M**ULTIPLICATION of Decimals is also perform'd the same way as Multiplication of whole Numbers; but to know the Value of the Product, observe this Rule.

Cut off, or separate by a Comma, or Prick, so many Decimal Places in the Product, as there are Places of Decimals in both Factors, *viz.* in the Multiplicand and Multiplier; which I shall farther explain in the following Examples.

Let 3.128 be multiply'd by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375: and because there were three Places of Decimals prick'd off in the Multiplicand, and two Places in the Multiplier, therefore you must prick off five Places of Decimals in the Product, as you may see by the Work.

$$\begin{array}{r}
 3.128 \\
 2.75 \\
 \hline
 15625 \\
 21875 \\
 6250 \\
 \hline
 8.59375
 \end{array}$$

Let

## 12 *Multiplication of DECIMALS. Part I.*

Let 79.25 be multiply'd by .459.

In this Example, because two Places of Decimals are prick'd off in the Multiplicand, and three in the Multiplier, therefore there must be five prick'd off in the Product.

$$\begin{array}{r}
 79.25 \\
 .459 \\
 \hline
 71325 \\
 39625 \\
 31700 \\
 \hline
 36.37575
 \end{array}$$

Let .135272 be multiply'd by .00425.

In this Example, because in the Multiplicand are six Decimal Places, and in the Multiplier five Places; therefore in the Product there must be eleven Places of Decimals; but when the Multiplication is finish'd, the Product is but 57490600, *viz.* only eight Places; therefore, in this Case, you must prefix three Cyphers before the Product Figures, to make up the Number of eleven Places: so the true Product will be .00057490600.

$$\begin{array}{r}
 .135272 \\
 .00425 \\
 \hline
 676360 \\
 270544 \\
 541088 \\
 \hline
 .00057490600
 \end{array}$$

# Chap. 5. Multiplication of DECIMALS. 13

*More Examples for Practice.*

.001472

.1045

7360

5888

14720

.0001538240

279.25

.445

139625

111700

111700

124.26625

4.443

15.98

35544

39987

22215

4443

70.99914

7.3564

.0126

441384

147128

73564

.09269064

.017532

347

122724

70128

52596

6.083604

32.0752

.0325

1603760

641504

962256

1.04244400

20.0291

35.45

1001455

801164

1001455

600873

710.031595

.75432

.0356

452592

377160

226296

.026853792

Can.

C

*Contracted Multiplication of Decimals.*

Because in Multiplication of Decimal Parts, and mix'd Numbers, there is no Need to express all the Figures of the Product, but in most Cases two, three, or four Places of Decimals will be sufficient; therefore, to contract the Work, observe this following

## R U L E.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whose Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which stands over the Figure you are then multiplying withal, and set down the first Figure of each particular Product directly one under the other: But yet a due Regard must be had to the Increase arising from the Figures on the Right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

*Example 1.* Let 2.38645 be multiply'd by 8.2175, and let there be only four Places retained in the Decimals of the Product.

First, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the rest of the Figures quite contrary to the usual Way, as in the following Work: Then begin to multiply, first the five which is left out, (only with regard to the Increase which must be carry'd from it, saying 8 times 5 is 40, carry 4 in your Mind, and say 8 times 4 is 32, and 4 I carry, is 36; and set down six, and carry 3, and proceed thro' the rest of the Figures, as in common Multiplication:



## Chap. 5. *Contracted Multiplication.* 15

Then begin to multiply with 2, saying 2 times 4 is 8, for which I carry 1, (because it is above 5) and say, 2 times 6 is 12, and 1 that I carry is 13; set down 3, and carry 1, and proceed thro' the rest of the Figures: Then multiply with 1, saying, once 6 is 6, for which carry 1, and say, once 8 is 8, and 1 is 9; set down 9, and proceed: Then multiply with 7, saying, 7 times 8 is 56, for which carry 6, (because it is above 55) and say, 7 times 3 is 21, and 6 that I carry is 27; set down 7, and carry 2, and proceed: Then multiply with 5, saying 5 times 3 is 15, for which carry 2, and say 5 times 2 is 10, and 2 I carry is 12, which set down, and add all the Products together; and the total Product will be 19.6107. See the Work.

$$\begin{array}{r}
 2.38645 \\
 5712.8 \\
 \hline
 19.0916 \\
 4773 \\
 239 \\
 167 \\
 12 \\
 \hline
 19.6107
 \end{array}$$

*Note,* That in multiplying the Figure left out every Time next the Reight Hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, &c.

I have here set down the Work of the last Example, wrought by the common Way, by which you may see both the Reason and Excellency of this Way, all the Figures on the Right Hand the Line being wholly omitted.

## Contracted Multiplication. Part I.

$$\begin{array}{r}
 2.38645 \\
 8.2175 \\
 \hline
 1193225 \\
 1670515 \\
 238645 \\
 477290 \\
 1909160 \\
 \hline
 19.6106152875
 \end{array}$$

*Example 2.* Let 375.13758 be multiply'd by 16.7324, so that the Product may have but four Places of Decimals.

First, set 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (because four Places of Decimals were to be prick'd off) and write all the rest of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the second Figure of the Multiplier 6, saying 6 times 8 is 48, for wick I carry 5, (in respect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; set down 5 and carry 3. and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and say 7 times 5 is 35, for which carry 4, and say 7 times 7 is 49, and 4 I carry is 53; set down 3 under the first, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and say 3 times 7 is 21, carry 2, and say 3 times 3 is 9, and 2 I carry is 11; set down 1 and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and say 2 times three is 6, for which I carry 1, and say 2 times 1 is 2, and 1 I carry is 3; set down 3; and 2 times 5 is 10; set down 0, and carry 1, and proceed as before. Then begin with 4 the last Figure of the Multiplier, and say 4 times 1 is 4, for which I carry nothing, because 'tis less than 5; then

## Chap. 5. *Contracted Multiplication.* 17

then say 4 times 5 is 20; set down 0, and carry 2, and proceed thro' the rest of the Figures of the Multiplicand. Then add all up together and the Product is 6276.9520. See the Work.

375.13758 the Multiplicand  
4237.61 the Multiplier revers'd.

---

37513758 the Product with 1.  
22508255 the Product with 6 increas'd with  $6 \times 8$ .  
2625963 the Product with 7 increas'd with  $7 \times 5$ .  
112541 the Product with 3 increas'd with  $3 \times 7$ .  
7503 the Product with 2 increas'd with  $2 \times 3$ .  
1500 the Product with 4 increas'd with 0.

---

6276.9520 the Product requir'd.

---

Let the same Example be repeated, and let only one Place in Decimals be prick'd off.

375.13758 the Multiplicand.  
4237.61 the Multiplier inverted.

---

37514 the Product by 1 with the Increase of  $1 \times 7$ .  
22508 the Product with 6 increas'd with  $6 \times 3$ .  
2626 the Product with 7 increas'd with  $7 \times 1$ .  
113 the Product with 3 increas'd with  $3 \times 5$ .  
7 the Product with 2 increas'd with  $2 \times 7$ .  
1 the Increase only of  $4 \times 3$ .

---

6276.9 the Product is the same as before

---

# 18      *Contracted Multiplication.*      Part I.

## *More Examples for Practice.*

Multiply 395.3756 by .75642; and prick off four Places in Decimals.

395.3756 the Multiplicand.  
 24657. the Multiplier revers'd.

---

2767629 the Product by 7 increas'd with  $7 \times 6$ .  
 197688 the Product by 5 increas'd with  $5 \times 5$ .  
 23722 the Product by 6 increas'd with  $6 \times 7$ .  
 1581 the Product by 4 increas'd with  $4 \times 3$ .  
 79 the Product by 2 increas'd with  $2 \times 5$ .

---

299.0699 the Product requir'd.

Let the same Example be repeated, and let there be only one Place of Decimals.

395.3756  
 24657.

---

2767 the Product by 7 increas'd with  $7 \times 3$ .  
 498 the Product by 5 increas'd with  $5 \times 5$ .  
 24 the Product by 6 increas'd with  $6 \times 9 + 6 \times 5$ .  
 2 the Increase of  $4 \times 9 + 4 \times 3$ .

---

299.1 the Product.

*Characters, and their Signification.*

*Note,* That this Mark  $+$  signifies Addition; as  $8+5$ , that is 8 more 5, or 8 added to 5; and  $8+3+7$  denotes these Numbers are to be added into one Sum.

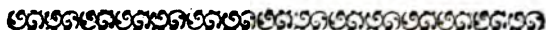
This Mark  $-$  signifies Subtraction, as  $9-4$  signifies that 4 is to be taken from 9.

This Mark  $\times$  signifies Multiplication, as  $7 \times 5$  signifies that 7 is to be multiply'd by 5.

This Mark  $\div$  signifies Division, as  $12 \div 4$  signifies 12 is to be divided by 4.

This Mark  $=$  signifies Equality, or Equation; that is, when  $=$  is placed between Numbers, or Quantities, it denotes them to be Equal, as  $7+5=12$ , that is, 7 more 5 is equal to 12; and  $15-7=8$ , that is, 15 less by 7, is equal to 8, or subtract 7 from 15 and there remains 8.

This Mark  $::$  is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion, thus,  $4:20::6:30$ , to be thus read, as 4 is to 20, so is 6 to 30.



C H A P. VI.

*Division of DECIMALS.*

**D**IVISION of Decimals is performed after the same Manner as Division of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty; for the resolving of which observe either of the following

R U L E S.

## R U L E S.

I. The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend which stands (or is to be suppos'd to stand) over the Unit's Place in the Divisor, at the first seeking.

II. When the Work of Division is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Divisor; for that Excess is the Number of Places which must be separated in the Question for Decimals: But if there be not so many Figures in the Quotient, as is the said Excess, that Deficiency must be supply'd with Cyphers in the Quotient, prefix'd before the significant Figures thereof, towards the Left Hand, with a Point before them; so shall you plainly discover the Value of the Quotient.

*These following Directions ought also to be carefully observ'd.*

If the Divisor consists of more Places than the Dividend, there must be a competent Number of Cyphers annex'd to the Dividend, to make it consist of as many (at least) or more Places of Decimals than the Divisor; for the Cyphers added must be reckon'd as Decimals.

Consider whether there be as many decimal Parts in the Dividend as there are in the Divisor; if there be not, make them so many, or more, by annexing of Cyphers.

In dividing of whole or mix'd Numbers, if there be a Remainder, you may bring down more Cyphers, and, by continuing your Division, carry the Quotient to as many Places of Decimals as you please.

These Things being consider'd, I shall proceed to the Practice of Division of Decimals, which I shall endeavour to explain in as familiar and easy a Method as possible.

*Example*

## Chap. 6. *Division of DECIMALS.* 21

*Example 1.* Let 48 be divided by 144.

In this Example the Divisor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, (*viz.* four) with a Point between them, and divide according to the usual way.

$$\begin{array}{r}
 144 \overline{) 48.0000(.3333} \\
 \underline{480} \phantom{000} \\
 480 \phantom{00} \\
 \underline{480} \phantom{0} \\
 48
 \end{array}$$

But, first, in seeking how often 144 in 48.0 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the first Place of Decimals; therefore the first Figure in the Quotient is in the first Place of Decimals: Or, by the second Rule, there being four Places of Decimals in the Dividend, and none in the Divisor; so the Excess of decimal Places in the Dividend, above that in the Divisor, is four; so that when the Division is ended, there must be four Places of Decimals in the Quotient. See the Work.

*Example 2.* Let 217.75 be divided by 65.

First, in seeking how oft 65 in 217 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the Unit's Place of the Dividend; therefore the first Figure in the Quotient will be Units, and all the rest Decimals. Or, by the second Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Divisor, therefore the Excess of decimal Places in the Dividend, above the Divisor, is two; so when the Division is ended, separate two Places in the Quotient, towards the Right Hand by a Point. See the Work.

$$65)217.75(3.35$$

$$\begin{array}{r} \dots \\ \hline 227 \\ 325 \\ \hline \dots \end{array}$$

*Example 3.* Let 267.15975 be divided by 13.25;

$$13.25)267.15975(20.163$$

$$\begin{array}{r} \dots \\ \hline 2159 \\ 8347 \\ 3975 \\ \hline \dots \end{array}$$

In this Example, 3, the Unit's Place of the Divisor, falls under 6, the Ten's Place of the Dividend; therefore (by the first Rule) the first Figure in the Quotient is Tens: Or, by the second Rule, the Excess of Decimal Places in the Dividend, above the Divisor, is three; there being five Places of Decimals in the Dividend, and but two in the Divisor, so there must be three Places of Decimals in the Quotient.

*Example 4.* Let 15.675159 be divided by 375.89.

$$375.89)15.675159(.0417$$

$$\begin{array}{r} \dots \\ \hline 63955 \\ 263669 \\ \hline 546 \end{array}$$



## Chap. 6. *Division of DECIMALS.* 23

In this Example, 5, in the Unit's Place of the Divisor, falls under 7, the second Place of Decimals in the Dividend; therefore (by the first Rule) the first Figure in the Quotient is in the second Place of Decimals; so that you must put a Cypher before the first Figure in the Quotient; and by the second Rule, the Excess of decimal Places in the Dividend above the Number of decimal Places in the Divisor is 4; for the decimal Places in the Dividend is 6, and the Number of Places in the Divisor but two; therefore there must be four Places of Decimals in the Quotient. But the Division being finish'd after the common Way, the Figures in the Quotient are but three; therefore you must prefix a Cypher before the significant Figures.

*Example 5.* Let 72.1564 be divided by .1347.

$$\begin{array}{r}
 .1347 \overline{) 72.1564(535.68} \\
 \underline{4806} \phantom{00} \\
 7654 \phantom{00} \\
 \underline{9190} \phantom{00} \\
 11080 \phantom{00} \\
 \underline{\phantom{00}304} \phantom{00}
 \end{array}$$

In this Example, the Divisor being a Decimal, the first Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) should fall under the Hundred's Place in the Dividend, and so the first Figure in the Quotient is Hundreds. And, by the second Rule, there being four Places of Decimals in the Dividend, and as many in the Divisor, so the Excess is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Division to two Places further; so I have two Places of Decimals. See the Work.

*Example*

## 24 Division of DECIMALS. Part I.

*Example 6.* Let .125 be divided by .0457.

$$.0457 \overline{) .1250000 (2.735}$$

$$\begin{array}{r} \text{....} \\ \hline 914^1 \\ \hline 3360 \\ 3199 \\ \hline 1610 \\ 1371 \\ \hline 3390 \\ 2285 \\ \hline 105 \end{array}$$

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to *Contracted Division*.

$$.00456 \overline{) .0000059791 (.00131.}$$

$$\begin{array}{r} \hline 1419 \\ 511 \\ \hline 55 \end{array}$$

## Chap. 7. *Extraction of the Square Root.* 37

Having pointed the given Number as before directed, seek what is the greatest square Number in the first Point 2, which is one; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; subtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and set on the Right Hand, making it 126; double the 1 in the Quotient; which makes 2; set 2 on the Left Hand for a Divisor, and ask how often 2 in 12, which will be 5 times; put 5 in the Quotient, and also on the Right Hand of the Divisor, making it 25; multiply (as in common Division) 25 by 5, and subtract the Product 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Resolvend; and double the 15 in the Quotient, it makes 30 for a new Divisor. Then seek how often 30 in 18, which you can't have; so that you must put 0 in the Quotient, and also on the Right Hand of the Divisor, and bring down the next Point, and it makes 18741 for another new Resolvend. Then seek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and also on the Right Hand of the Divisor; multiply and subtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a time, to the Remainders, and so prosecute the Work to what Number of decimal Parts you please; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Divisor: Then seek how often 3012 in 7050, (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and also on the Right Hand of the Divisor, and multiply and subtract before, and the Remainder will be as 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the square Root of the given Number is 1506.23, which being squared, or multiply'd by itself, and the last Remainder added, will make the given Number as follows.

### 38 *Extraction of the Square Root. Part I.*

$$\begin{array}{r}
 1506.23 \\
 1506.23 \\
 \hline
 451869 \\
 301246 \\
 903738 \\
 7531150 \\
 150623 \\
 \hline
 2268728.8129 \\
 \text{The Remainder add—} 12.1871 \\
 \hline
 \text{Proof } 2268741.0000
 \end{array}$$

*Some more Examples for Practice.*

*Example 1.*      $\sqrt{7596796(2756.228 \text{ Root.})}$

$$\begin{array}{r}
 4 \\
 \hline
 47 \overline{)359} \\
 \underline{329} \\
 545 \overline{)3067} \\
 \underline{2725} \\
 5506 \overline{)34296} \\
 \underline{33036} \\
 55122 \overline{)126000} \\
 \underline{110244} \\
 551242 \overline{)1575600} \\
 \underline{1102484} \\
 5512448 \overline{)47311600} \\
 \underline{44099584} \\
 3212016
 \end{array}$$

*Example*

# Chap. 7. *Extraction of the Square Root.* 39

*Example 2.*  $\overset{\cdot}{7}\overset{\cdot}{5}\overset{\cdot}{1}\overset{\cdot}{4}\overset{\cdot}{1}\overset{\cdot}{7}\overset{\cdot}{.}\overset{\cdot}{5}\overset{\cdot}{7}\overset{\cdot}{4}\overset{\cdot}{5}$  (866.4 Root.  
64

$$\begin{array}{r} 166)11114 \\ \underline{996} \end{array}$$

$$\begin{array}{r} 1726)11817 \\ \underline{10356} \end{array}$$

$$\begin{array}{r} 17328)146157 \\ \underline{138624} \end{array}$$

$$\begin{array}{r} 173364)753345 \\ \underline{693456} \end{array}$$

$$\underline{\underline{59889}}$$

If the given Number be a mix'd Number, *viz.* consisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, &c. that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

## 40 *Extraction of the Square Root. Part I.*

*Example 3.* Let 656714.37512 be given, to find the square Root.

$$\begin{array}{r}
 656714.375120(810.379 \text{ Root.} \\
 \underline{64} \\
 161)167 \\
 \underline{161} \\
 16203)61437 \\
 \underline{48609} \\
 162067)1282851 \\
 \underline{1134469} \\
 1620749)14838220 \\
 \underline{14586741} \\
 \text{Remains } 251479
 \end{array}$$

In this Example there are five Places of Decimals ; therefore put a Cypher to it, to make it even, that so there may a Point fall upon 4, the Unit's Place.

### *To find the Square Root of a Fraction.*

If it be a decimal Fraction, the Work differs nothing from the Examples aforegoing, only you must be mindful to point your given Number aright ; for (as was before directed) the Number of Places must always be made even, and then begin to point at the Right Hand, as in whole Numbers.

If it be a vulgar Fraction, it must be reduc'd to a Decimal, by the first Rule of the second Chapter.

I shall give an Example or two in each Case, and so conclude this Chapter.

Let

## Chap. 7. *Extraction of the Square-Root.* 41

Let .125 be a decimal Fraction given, whose square Root is requir'd ; and let it be requir'd to have four Places of Decimals in the Root.

$$\begin{array}{r}
 .12500000(.3535 \\
 \underline{9} \\
 65)350 \\
 \underline{325} \\
 703)2500 \\
 \underline{2109} \\
 7065)39100 \\
 \underline{35325} \\
 3775
 \end{array}$$

In this Example there must be five Cyphers annex'd, because two Places in the Square make but one in the Root.

Let the square Root of .00715 be requir'd.

$$\begin{array}{r}
 .007150(.084 \\
 \underline{64} \\
 164)750 \\
 \underline{656} \\
 94
 \end{array}$$

In this a Cypher is added to make the Place even.

## 42 *Extraction of the Square Root. Part I.*

Let  $\frac{7}{8}$  be a vulgar Fraction given, whose square Root is requir'd.

$$\begin{array}{r}
 8 \overline{) 7000} \\
 \underline{64} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{40} \\
 \dots
 \end{array}
 \qquad
 \begin{array}{r}
 .87500000(.9354 \\
 81 \\
 \underline{18} \overline{) 650} \\
 549 \\
 \underline{1865} \overline{) 10100} \\
 9325 \\
 \underline{18704} \overline{) 77500} \\
 74816 \\
 \underline{2684}
 \end{array}$$

Reduce this  $\frac{7}{8}$  to a Decimal, it makes .875; to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

Let  $\frac{1}{2}$  be a vulgar Fraction, whose square Root is requir'd.

$$\begin{array}{r}
 96 \overline{) 0} 3.000000 \quad (.00312500(.0559 \text{ Root.} \\
 288 \\
 \underline{120} \\
 96 \\
 \underline{240} \\
 192 \\
 \underline{480} \\
 480 \\
 \underline{\dots}
 \end{array}
 \qquad
 \begin{array}{r}
 25 \\
 \underline{105} \overline{) 625} \\
 525 \\
 \underline{1109} \overline{) 10000} \\
 9981 \\
 \underline{19}
 \end{array}$$



## Chap. 8. *Extraction of the Cube Root.* 43

In extracting the Root of this, because the first Point consists of Cyphers, there must be a Cypher put first in the Quotient.

To prove this Rule, square the Root, and to the Product add the Remainder, as was before directed. To square a Number, is to multiply it by itself; and to cube it, is to multiply the Square of the Number by the Number itself.



## CHAP. VIII.

### *Extraction of the CUBE ROOT.*

**T**O extract the Cube Root, is nothing else but to find such a Number, as being first multiply'd into itself, and then into that Product, produceth the given Number; which to perform, observe these following Directions.

1<sup>st</sup>, You must point your given Number, beginning with the Unit's Place, and make a Point, or Dot, over every third Figure towards the Left Hand.

2<sup>dly</sup>, Seek the greatest Cube Number in the first Point, towards the Left Hand, putting the Root thereof in the Quotient, and the said Cube Number under the first Point, and subtract it therefrom, and to the Remainder bring down the next Point, and call that the Resolvend.

3<sup>dly</sup>, Triple the Quotient, and place it under the Resolvend; the Unit's Place of this under the Ten's Place of the Resolvend; and call this the triple Quotient.

4<sup>thly</sup>, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of  
this

#### 44 *Extraction of the Cube Root. Part I.*

this under the Ten's Place of the triple Quotient, and call this the triple Square.

5<sup>thly</sup>, Add these two together, in the same Order as they stand, and the Sum shall be the Divisor.

6<sup>thly</sup>, Seek how often the Divisor is contain'd in the Resolvend, rejecting the Unit's Place of the Resolvend, (as in the Square Root) and put the Answer in the Quotient.

7<sup>thly</sup>, Cube the Figure last put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Resolvend.

8<sup>thly</sup>, Multiply the Square of the Figure last put in the Quotient, into the triple Quotient, and place the Product under the last, one Place more to the Left Hand.

9<sup>thly</sup>, Multiply the triple Square by the Figure last put in the Quotient, and place it under the last, one Place more to the Left Hand.

10<sup>thly</sup>, Add the three last Numbers together, in the same Order as they stand, and call that the Subtrahend.

Lastly, Subtract the Subtrahend from the Resolvend; and if there be another Point, bring it down in the Remainder, and call that a new Resolvend, and proceed in all respects as before.

*Example*

## Chap. 8. *Extraction of the Cube Root.* 45

*Example* 1. Let 314432 be a Cubick Number, whose Root is required.

314432 (68 Root  
216

---

98432 Resolvend.

---

18 Triple Quotient of 6.

108 Triple Square of the Quotient 6.

---

1098 Divisor.

---

512 Cube of 8, the last Figure of the Root.

1152 The Square of 8, by the triple Quotient.

864 The triple Square of the Quotient 6 by 8.

---

98432 The Subtrahend.

.....

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table. (*Page* 34.) you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolvend. Then triple the Quotient 6, it makes 18, which write down the Unit's Place, 8, under 3, the Ten's Place of the Resolvend. Then square the Quotient 6, and triple that Square, and it makes 108, which write under the triple Quotient, one Place toward the Left Hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contain'd in the Resolvend, (rejecting the Unit's Place thereof) that is, how often 1098 in 9843, which is 8 times;

## 46 *Extraction of the Cube Root. Part I.*

8 times ; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18 ; the Product is 1152 ; set this under the Cube of 8, the Units of this under the Tens of that. Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864 ; set this down under the last Product, a Place more to the Left Hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is called the Subtrahend ; which being subtracted from the Resolvend, the Remainder is nothing ; which shews the Number to be a true cubick Number, whose Root is 68 ; that is, if 68 be cub'd, it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624 ; and this Product, multiply'd again by 68, the last Product is 314432, which shews the Work to be right.

|            |                                       |
|------------|---------------------------------------|
|            | 68                                    |
|            | 68                                    |
|            | <hr style="width: 50px; margin: 0;"/> |
| The Work   | 544                                   |
|            | 408                                   |
|            | <hr style="width: 50px; margin: 0;"/> |
|            | 4624                                  |
|            | 68                                    |
|            | <hr style="width: 50px; margin: 0;"/> |
|            | 36992                                 |
|            | 27744                                 |
|            | <hr style="width: 50px; margin: 0;"/> |
| The Proof. | 314432                                |

*Example 2.* Let the Cube Root of 5735339 be requir'd.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which

## Chap.8. *Extraction of the Cube Root.* 47

which (by the little Table, *page 34.*) you will find to be 1; which place under 5, and 1, the Root thereof, in the Quotient; and subtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Resolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which set one under another, in their Order, and added, makes 33 for the Divisor. Seek how often the Divisor in the Resolvend, and proceed as in the last Example.

5735339(179 Root.

1

---

4735 Resolvend.

---

3 The Triple of the Quotient 1, the first Figure.  
3 The triple Square of the Quotient 1.

---

33 The Divisor.

---

343 The Cube of 7, the second Figure of the Root.  
147 The Square of 7, multipl. in the triple Quot. 3.  
21 The triple Squ. of the Quot. multiply'd by 7.

---

3913 The Subtrahend.

---

822339 The new Resolvend.

---

51 The Triple of the Quot. 17, the two first Fig.  
867 The triple Square of the Quotient 17.

---

8721 Divisor.

---

729 The Cube of 9, the last Figure of the Root.  
4131 The Square of 9, multipl. by the trip. Quot. 51.  
7803 The triple Square of the Quotient 867 by 9.

---

822339 The Subtrahend

....

## 48 *Extraction of the Cube Root. Part I.*

In this Example, 33, the first Divisor seems to be contain'd more than seven times in 4735, the Resolvend; but if you work with 9, or 8, you will find, that the Subtrahend will be greater than the Resolvend.

### *Some more Examples for Practice.*

32461759(319 The Root.

27

5461 Resolvend.

9 The triple of 3.

27 The triple Square of 3.

279 the Divisor.

1 The Cube of 1, the second Figure.

9 The triple Quotient, by the Square of 1.

27 The triple Square, multipl. by 1, the 2d Figure.

2791 The Subtrahend.

2670759 A new Resolvend.

93 The Triple of 31.

2883 The triple Square of 31.

28923 The Divisor.

729 The Cube of 9, the last Figure.

7533 The Square of 9, by 93 the triple Quotient.

25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

.....

# Chap. 6. Division of DECIMALS.

25

Let it be divided by 282.

$$282)1.0000000(.0035461 \text{ ferè.}$$

$$\begin{array}{r} \text{.....} \\ \hline 1540 \\ 1300 \\ 1720 \\ 280 \end{array}$$

$$.325)4.000000(1.2307$$

$$\begin{array}{r} \text{....} \\ \hline 750 \\ 1000 \\ 2500 \\ \hline 225 \end{array}$$

$$.042)495.00000(11785.71$$

$$\begin{array}{r} \text{.....} \\ \hline 75 \\ 330 \\ 360 \\ 240 \\ 300 \\ 60 \\ \hline 18 \end{array}$$

*Division of DECIMALS contracted.*

- **I**N Division of Decimals, the common Way, when the Divisor hath many Figures, and it is requir'd to continue the Division till the Value of the Remainder be but small, the Operation will sometimes be large and tedious, but may be excellently contracted by the following Method.

*The R U L E.*

By the first Rule of this Chapter, (page 20.) find what is the Value of the first Figure in the Quotient; then, by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the Left Hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Divident as will answer them; and in dividing, omit one Figure of the Divisor at each following Operation. A few Examples will make it plain.

*Example 1.* Let 721.17562 be divided by 2.257432; and let there be three Places of Decimals in the Quotient.

2.25743



2.25743)721.17562(319.467

..... 677229

43946

22574

21372

20317

1055

903

152

135

17

15

2

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend, and it is requir'd. that three Places of Decimals be in the Quotient, so there must be six Places in all, that is, three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how oft the Divisor in the Dividend, and the Answer is three times; put 3 in the Quotient, and multiply and subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how often the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and subtract, and the next Remainder is 21372. Then prick off the 4 in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient; multiply thus, saying 9

D 2

times

times 4 is 36, for which I carry 4, (in respect of the 4 last prick'd off) and 9 times 7 is 63, and 4 is 67; set down 7, and carry 6, and so proceed till the Division be finish'd, always respecting the Increase made from the Figures prick'd off. Observe the Work which will better inform you than many Words.

$$2.25743 \overline{) 721.47562(319.467}$$

|        |       |
|--------|-------|
| 677229 |       |
| 43046  | 6     |
| 22574  | 3     |
| 21372  | 32    |
| 20316  | 87    |
| 1055   | 450   |
| 902    | 972   |
| 152    | 4780  |
| 135    | 4458  |
| 17     | 03220 |
| 15     | 80201 |
| 1      | 23019 |

I have set down the Work of this last Example at large, according to the common Way, that thereby the Learner may see the Reason of the Rule, all the Figures on the Right Side the perpendicular Line being wholly omitted.

*Example*

# Chap. 6. Contracted Division.

29

*Example 2.* Let 5171.59165 be divided by 8.758615; and let it be requir'd, that four Places of Decimals be prick'd off in the Quotient.

$$8.758615 \overline{) 5171.59165 (590.4577}$$

$$\begin{array}{r} 43793075 \\ \hline \end{array}$$

$$\begin{array}{r} 7922841 \\ \hline \end{array}$$

$$\begin{array}{r} 7882754 \\ \hline \end{array}$$

$$\begin{array}{r} 40087 \\ \hline \end{array}$$

$$\begin{array}{r} 35034 \\ \hline \end{array}$$

$$\begin{array}{r} 5053 \\ \hline \end{array}$$

$$\begin{array}{r} 4379 \\ \hline \end{array}$$

$$\begin{array}{r} 674 \\ \hline \end{array}$$

$$\begin{array}{r} 613 \\ \hline \end{array}$$

$$\begin{array}{r} 61 \\ \hline \end{array}$$

$$\begin{array}{r} 61 \\ \hline \end{array}$$

...

In this Example, I can't have 8, the first Figure in the Divisor, in 5, the first Figure of the Dividend; so that the Unit's Place of the Divisor falls under the Hundred's Place of the Dividend; so that there will be seven Figures in the Quotient, that is, three of whole Numbers, and four of Decimals; therefore there must be 7 Figures in the Divisor, (because the Number of Places in the Divisor and Quotient will be equal) and there must be eight Places in the Dividend; so that I cut off the Figure 5 with a Dash, as useless. Thus having proportion'd the Dividend to the Divisor, and both to the Number of Places or Figures desir'd in the Quotient, I proceed to divide as before, saying how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and subtract,

D 3

and

and the Remainder is 7922841. Then I prick off the first Figure in the Divisor, 5, and seek how often the remaining Figures of the Divisor in the aforesaid Remainder, which I find nine times; put 9 in the Quotient, and multiply thereby, saying 9 times 5 (the Figure prick'd off) is 45, for which I carry 5, and say 9 times 1 is 9, and 5 I carry is 14; set down 4, and carry 1, and proceed to multiply the rest of the Figures, and subtract, and the Remainder will be 40087. Then prick off the Figure 1, and seek how often 87586 in the Remainder 40087, the Answer will be 0; so put 0 in the Quotient, and prick off the Figure 6, and seek how often 8758 in 40087, which will be four times; put 4 in the Quotient, and multiply, saying 4 times 6 (the Figure last prick'd off) is 24, for which I carry 2, and say 4 times 8 is 32, and 2 I carry is 34; set down 4, and carry 3; multiply the rest of the Figures, and subtract as before, and so proceed after the same manner, until all the Figures of the Divisor be prick'd off, to the last Figure. See the Work.

*Example 3.* Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example, 7, the Unit's Place of the Divisor, falls under 1, the first Place of Decimals; therefore the first Figure of the Quotient is in the first Place of Decimals; so the Quotient will be all Decimals. Then, because the Quotient Figures, and the Figures of the Divisor will be of an equal Number, dash off the 43 in the Divisor, and the 7 in the Dividend, as useless, and divide as before.

$$\begin{array}{r}
 217.35 \overline{) 25.13617(.11564} \\
 \underline{21735} \phantom{00} \\
 3401 \phantom{00} \\
 \underline{2174} \phantom{00} \\
 1227 \phantom{00} \\
 \underline{1087} \phantom{00} \\
 140 \phantom{00} \\
 \underline{130} \phantom{00} \\
 10 \phantom{00} \\
 \underline{8} \phantom{00} \\
 2
 \end{array}$$

Altho' I have hitherto given Directions for proportioning the Divisor and Dividend, so as to bring into the Quotient what Number of Decimals you please, yet there is no absolute Necessity for it; but you may carry on your Division to what Degree you please, before you begin to prick off the Figures of the Divisor, in order to contract the Work, as in the following Examples, where it is not requir'd to prick off any determinate Number of Decimals, but it may be done according to Discretion.

2.756756

2-756756)7414-76717(2689.67118

.....

---

5513512

---

19012551  
16540536

---

24720157  
22054048

---

2666109  
2481080

---

185029  
165405

---

19624  
19297

---

327  
276

---

51  
28

---

23  
22

---

3

12.34254

# Chap 7. *Extraction of the Square Root.* 33

12.34254)514.75498(41.705757

... 4937016

2105338

1234254

871084

863978

7106

6171

935

864

71

62

9

8

1

1

## CH A P. VII.

### *Extraction of the SQUARE ROOT.*

*If a Square Number be given ;*

**T**O find the Root thereof, that is, to find out such a Number, as being multiply'd into itself, the Product shall be equal to the Number given ; such Operation is call'd, *The Extraction of the Square Root ;* which to do, observe the following Directions.

### 34 *Extraction of the Square Root. Part. I.*

1<sup>st</sup>, You must point your given Number; that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and so upon every second Figure throughout.

2<sup>dly</sup>, Then seek the greatest square Number in the first Point towards the Left Hand, placing the square Number under the first Point, and the Root thereof in the Quotient, and subtract the said square Number from the first Point, and to the Remainder bring down the next Point, and call that the Resolvend.

3<sup>dly</sup>, Then double the Quotient, and place it, for a Divisor, on the Left Hand of the Resolvend; and seek how often the Divisor is contained in the Resolvend, (reserving always the Unit's Place) and put the Answer in the Quotient, and also on the Right Hand Side of the Divisor; then multiply by the Figure last put in the Quotient, and subtract the Product from the Resolvend, (as in common Division) and bring down the next Point to the Remainder, (if there be any more) and proceed as before.

### *A TABLE of Squares and Cubes, and their Roots.*

| Root | 1 | 2 | 3  | 4  | 5   | 6   | 7   | 8   | 9   |
|------|---|---|----|----|-----|-----|-----|-----|-----|
| Squ. | 1 | 4 | 9  | 16 | 25  | 36  | 49  | 64  | 81  |
| Cub. | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 521 | 729 |

*Example 1.* Let 4489 be a Number given, and let the square Root thereof be requir'd.

4489.67  
36

127.889 Resolvend.  
889 Product.

...

First,



## Chap. 7. *Extraction of the Square Root.* 35

First, Point the given Number, as before directed; then, by the little Table foregoing, seek the greatest square Number in 44, (the first Point to the Left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the Right Hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12, which place on the Left Hand for a Divisor, and seek how often 12 in 88, (reserving the Unit's Place) the Answer is 7 times; which put in the Quotient, and also on the Right Hand Side of the Divisor, and multiply 127 by 7, as in common Division, and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finish'd; and the square Root of 4489 is 67; which Root if you multiply by itself; that is 67 by 67, the Product will be 4489, equal to the given square Number, and prove the Work to be right.

*Example 2.* Let 106929 be a Numbtr given, and let the square Root thereof be requir'd.

$$\begin{array}{r}
 106929(.327 \\
 9 \phantom{000000} \\
 \hline
 62)169 \text{ Resolvend.} \\
 124 \text{ Product.} \\
 \hline
 47)4529 \text{ Resolvend.} \\
 4529 \text{ Product.} \\
 \hline
 \end{array}$$

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10, (the first Point) which by the little

### 36 *Extraction of the Square Root. Part I.*

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then subtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Resolvend; then double the Quotient 3, and it makes 6, which place on the Left Hand of the Resolvend for a Divisor, and seek how often 6 in 16; the Answer is twice; put two in the Quotient, and also on the Right Hand of the Divisor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124: which subtract from the Resolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Resolvend. Then double the Quotient 32, and it makes 64, which place on the Left Side of the Resolvend for the Divisor, and seek how oft 64 in 452, which you'll find 7 times; put seven in the Quotient, and also on the Right Hand of the Divisor, making it 647, which multiply'd by the 7 in the Quotient, it makes 4529, which subtracted from the Resolvend, there remains nothing. So 327 is the Square Root of the given Number.

*Example 3.* Let 2268741 be a square Number given, the Root whereof is requir'd.

$$\begin{array}{r}
 2268741 \\
 \sqrt{\phantom{000000}} \\
 1 \\
 \hline
 25 \overline{) 126} \\
 \underline{125} \phantom{00} \\
 3006 \overline{) 18741} \\
 \underline{18036} \phantom{00} \\
 30122 \overline{) 70500} \\
 \underline{60244} \phantom{00} \\
 301243 \overline{) 1025600} \\
 \underline{908729} \phantom{00} \\
 \hline
 \text{Remains } 121871
 \end{array}$$

Having

Chap. 8. *Extraction of the Cube Root.* 49

84604519 The Root  
64

20604 Resolvend,

12 The Triple of 4.  
48 The triple Square of 4.

492 The Divisor.

27 The Cube of 3.  
108 The Square of 3, by the triple Quotient.  
144 The triple Square by 3.

15507 The Subtrahend.

5097519 The Resolvend.

129 The Triple of 43.  
5547 The triple square of 43.

55599 The Divisor.

729 The Cube of 9.  
10449 The Square of 9 by 129.  
49923 The triple Square by 9.

5097519 The Subtrahend.

.....

## 50 *Extraction of the Cube Root. Part I.*

259697989(638  
216

---

43697 Resolvend.

18 The Triple of 6.  
108 The Triple Square of 6.

---

1098 The Divisor.

---

27 The Cube of 3, the 2d Figure.  
162 The triple Square of 3 by 18.  
324 The triple Square 108 by 3.

---

34047 The Subtrahend.

---

9650989 Resolvend.

189 The Triple of 63.  
11907 The triple Square of 63.

---

119259 The Divisor.

---

512 The Cube of 8.  
12096 The Square of 8 by 189.  
95256 The triple Square 11907 by 8.

---

9647072 The Subtrahend.

---

3917 The Remainder.

# **Chap. 8. Extraction of the Cube Root. 51**

25917056(295.9  
8

17917 The Resolvend.

6 The Triple of 2.

12 The triple Square of 2.

126 The Divisor.

729 The Cube of 9, the 2d Figure.

486 The Square of 9 by 6.

108 The triple Square by 9.

16389 The Subtrahend.

1528056 The Resolvend.

87 The Triple of 29.

2523 The triple Square of 29.

25317 The Divisor.

125 The Cube of 5, the 3d Fig.

2175 The Square of 5 by 87.

12615 The triple Square by 5.

1283375 The Subtrahend.

244681000 The Resolvend.

885 The Triple of 295.

261075 The triple Square of 295.

2611635 The Divisor.

729 The Cube of 9, the last Figure.

71685 The Square of 9 by 885.

2349675 The triple Square by 9.

235685079 The Subtrahend.

8995921 The Remainder.

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Resolvend; by which means I bring one Place of Decimals. And so you may proceed to more decimal Places at Pleasure, by annexing three Cyphers to the next Remainder, and carrying on the Work as before.

## 52 *Extraction of the Cube Root. Part I.*

22069810125(2805  
8

14069 Resolvend.

6 The Triple of 2.

12 The triple Square of 2.

126 The Divisor.

512 The Cube of 8.

384 The Square of 8 by 6.

96 The triple Square by 8.

13952 The Subtrahend.

117810125 New Resolvend.

84 The Triple of 28.

2352 The triple Square of 28.

23604 Divisor.

840 The Triple of 280.

235200 The triple Square of 280.

2352840 New Divisor.

125 Cube of 5.

21000 Square of 5 by 840.

1176000 Triple Square by 5.

117810125 Subtrahend.

.....

In this Example 13952 being subtracted from the Resolv. 14069, the Remainder is 117 ; to which bring down 810, the third Point, and it makes 117810, for a new Resolv. and the next Divisor is 23604, which you cannot have in the said Resolv. ( the Unit's Place being rejected ) ; so you must put 0 in the Quotient, and seek a new Divisor ( after you have brought down your last Point to the Resolvend ) ; which new Divisor is 2352840 ; which you will find to be contained 5 times. So proceed to finish the rest of the Work.

# Chap. 8. *Extraction of the Cube Root.* 53

93759.575070(45.42  
64

29759 The Resolvend.

12 The Triple of 4, the first Figure.  
48 The triple Square of 4.

492 The Divisor.

125 The Cube of 5, the 2d Figure.  
300 The Square of 5 by 12, the triple Quotient.  
240 The triple Square by 5.

27125 The Subtrahend.

2634575 The Resolvend.

135 The Triple of 45.  
6075 The triple Square of 45.  
60885 The Divisor.

64 The Cube of 4.  
2160 The Square of 4 by 135.  
24300 The triple Square by 4.  
2451664 The Subtrahend.

182911070 The Resolvend.

1362 The Triple of 45.4.  
618348 The triple Square of 45.4.  
6184842 The Divisor.

8 The Cube of 2.  
5448 The Square of 2 by 1362.  
236696 The triple Square of 2 by 2.

123724088 The Subtrahend.

59186982 The Remainder.

## 54 *Extraction of the Cube Root. Part I.*

In extracting the Cube Root of a mix'd Number, always observe to make the decimal Part to consist of either three, six, nine, &c. Places, that is, always to consist of even Points, as in the last Example, where the decimal Places were five; to which I annex'd a Cypher to make up six, and so I proceed to point it; and by that means I have a Point falls upon the Unit's Place of whole Numbers, which you must always observe.

### *To extract the CUBE ROOT out of a Fraction.*

This is the same to do as in whole Numbers, observe but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal must always consist of three, six, nine, &c. Places; and if it be not so, it must be made so, by annexing of Cyphers, as is above said.

If the Cube Root of a vulgar Fraction be requir'd, you must first reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

*Example*



## Chap. 8. *Extraction of the Cube Root.* 55

*Example 1.* Let the Cube Root of .401719179 be required.

.401719179(.737 Root.  
343

58719 Resolvend.

21 Triple of 7.  
147 Triple Square of 7.

1491 Divisor.

27 Cube of 3.  
189 Square of 3 by 21.  
441 Triple Square by 3.

46017 Subtrahend.

12702179 Resolvend.

219 Triple of 73.  
15987 Triple Square of 73.

160089 Divisor.

343 Cube of 7.  
10731 Square of 7 by 219.  
111909 Triple Square by 7.

11298553 Subtrahend.

1406236 Remainder.

*Example*

## 56 *Extraction of the Cube Root. Part I.*

*Example 2.* Let the Cube Root of .0001416 be requir'd.

.000141600(.052 Root.  
125

16600 Resolvend.

15 The Triple of 5.

75 Triple Square of 5.

765 Divisor.

8 Cube of 2.

60 The Square of 2 by 15.

150 Triple Square by 2.

15608 Subtrahend.

992 Remainder.

*Example 3.* Let  $2\frac{3}{6}$  be a vulgar Fraction, whose Cube Root is requir'd.

By the first Rule of Chapter II. reduce the vulgar Fraction to a Decimal,

276)5.000000000(.018115942

2240

320

440

1640

2600

1160

560

8

.01812

# **Chap. 8. *Extraction of the Cube Root.* 57**

.018115942(.262 Root.  
8

---

10115 Resolvend.

---

216 Triple of 2.  
12 Triple Square of 2.

---

126 Divisor.

---

216 Cube of 6.  
216 Square of 6 by the Triple of 2.  
72 The triple Square by 6.

---

9576 Subtrahend.

---

539942 Resolvend.

---

78 Triple of 26.  
2028 Triple Square of 26.

---

20358 Divisor.

---

8 Cube of 2.  
312 Square of 2 by 78.  
4056 Triple Square 2028 by 2.

---

408728 Subtrahend.

---

131214 Remainder.

You may prove the Truth of the Work, by cubing the Root found, as was shew'd in the first Example; and if any thing remains, add it to the said Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will

## 58 *Extraction of the Cube Root. Part I.*

I will shew the Proof of the fifth Example, (*pag.* 50.) the given Number being 259697989, whose Root is 638, it being a surd Number, there remains 3917.

$$\begin{array}{r}
 638 \\
 638 \\
 \hline
 5104 \\
 1914 \\
 3828 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{The Square } 407044 \\
 638 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3256352 \\
 1221132 \\
 2442264 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{The Cube } 259694072 \\
 \text{The Remainder add } 3917 \\
 \hline
 \end{array}$$

Proof equal to the given Number 259697989

~~~~~

C H A P. IX.

Multiplication of Feet, Inches, and Parts.

IN the multiplying of Feet, Inches, &c. I shall endeavour to lay down such easy and familiar Rules, as may easily be understood by the meanest Capacity.

Example

Chap. 9. *Multiplication of Feet, &c.* 59

Example 1. Let 7 Feet 9 Inches be multiply'd by 3 Feet 6 Inches.

$$\begin{array}{r}
 \text{F. I.} \\
 7 \quad 9 \\
 3 \quad 6 \\
 \hline
 23 \quad 3 \text{ Pts.} \\
 3 \quad 10 \quad 6 \\
 \hline
 27 \quad 1 \quad 6
 \end{array}$$

First, Multiply 9 Inches by 3, saying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; set down 3 under Inches, and carry 2 to the Feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the Feet.

Then begin with 6 Inches, saying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; set down 6 Parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which set down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiply'd by 9 Feet 8 Inches.

$$\begin{array}{r}
 \text{F. I.} \\
 75 \quad 7 \\
 9 \quad 8 \\
 \hline
 680 \quad 3 \\
 50 \quad 4 \quad 8 \\
 \hline
 730 \quad 7 \quad 8
 \end{array}$$

First, Multiply by 9 Feet, saying, 9 times 7 is 63, which is 5 Feet 3 Inches; set down 3, and carry 5, saying, 9 times 5 is 45, and 5 I carry is 50; set down 0, and carry 5, saying, 9 times 7 is 63, and 5

60 *Multiplication of Feet, &c. Part I.*

is 68 ; set down 68, and proceed to multiply by 8 Inches, saying, 8 times 7 is 56 ; the Twelves in 56 are four times ; and 8 remains : set 8 a Place to the Right-hand, and carry 4 : Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains ; set down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet 7 Inches 8 Parts.

I will repeat the last Example again, and shew another Way to work it, which, I think, is better, and more expeditious, when there are more Figures than one in the Feet ; thus,

$$\begin{array}{r}
 \text{F. I.} \\
 75 \quad 7 \\
 9 \quad 8 \\
 \hline
 680 \quad 3 \\
 25 \quad 2 \quad 4 \\
 25 \quad 2 \quad 4 \\
 \hline
 730 \quad 7 \quad 8
 \end{array}$$

Multiply by 9 Feet, first, as above directed ; then, instead of multiplying by 8 Inches, let the 8 Inches be parted into such aliquot or even Parts of a Foot, as you find to be contain'd in that Figure ; if you take such Parts of the Multiplicand, and add them to the former Product, the Sum will give the Answer : Thus, 8 Inches may be parted into 4, and 4, because 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and set it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the same as before ; thus, say how often 3 in 7, which is twice ; set down 2 ; then, because twice 3 is 6, say, 6 out of 7, and there remains 1, for which you must add 10 to the 5, and it makes 15 ; then the Threes in 15 are 5 times ; set down 5 ; and, because 3 times 5 is 15, there is 0 remains. Then go

Chap. 9. *Multiplication of Feet, &c:* 61

to the 7 Inches, saying, the Threes in 7 are twice; set down 2 in the Inches; and because twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet 2 Inches 4 Parts; which set down again, and add all together, the Sum is 730 Feet 7 Inches 8 Parts; the same as before.

Example 3. Let 97 Feet 8 Inches be multiply'd by 8 Feet 9 Inches.

$$\begin{array}{r}
 \text{F. I.} \\
 97 \quad 8 \\
 8 \quad 9 \\
 \hline
 781 \quad 4 \\
 48 \quad 10 \\
 24 \quad 5 \\
 \hline
 854 \quad 7
 \end{array}$$

Begin, first, to multiply by 8 Feet, saying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; set down 4 Inches, and carry 5, saying, 8 times 7 is 56, and 5 I carry is 61; set down 1, and carry 6, saying, 8 times 9 is 72, and 6 I carry is 78, which set down: Then, instead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3; 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet, 8 Inches, which is 48 Feet 10 Inches; and because 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work, as above.

62 *Multiplication of Feet, &c. Part I.*

Example 4. Let 75 Feet 9 Inches be multiply'd by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
<hr/>		
525		
75		
25	3	P.
18	11	3
8	6	
4	3	
<hr/>		
1331	11	3
<hr/>		

In this Example, because there are more than 12 Feet in the Multiplier, therefore I first multiply the 75 by 17 Feet; then because the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches (not meddling with the 7 Inches, because that was multiply'd into the 9 before); then add all these together, and the Sum is 1331 Feet 11 Inches 3 Parts,

Example

Chap. 9. Multiplication of Feet, &c. 63

Example 5. Let 87 Feet 5 Inches be multiply'd by 35 Feet 8 Inches.

F.	I.	
87	5	
35	8	
435		
261		P.
29	1	8
29	1	8
11	8	0
2	11	0

3117 10 4

Work here as in the last Example. After you have multiply'd the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches and set it down twice. Thus the 3d Part of 87 Feet 5 Inches is 29 Feet 1 Inch 8 Parts; set this down twice: Then the aliquot Parts of 5 Inches are 4 and 1, that is, a 3d Part and a 12th Part; therefore take a 3d Part of 35, which is 11 Feet 8 Inches, and a 12th Part of 35 is 2 Feet 11 Inches; set all these one under another, and add them together, and the Sum is 3117 Feet 10 Inches 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiply'd by 48 Feet 11 Inches,

F.	I.	
259	2	
49	11	
2072		
1036		
129	7	P.
86	4	8
21	7	2
8	0	0
12677	6	10
G	2	

First

64 *Multiplication of Feet, &c. Part I.*

First, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, an half, a third, and a twelfth; therefore take the half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet 4 Inches 8 Parts, and the twelfth Part of 259 Feet 2 Inches is 21 Feet 7 Inches 2 Parts; or (because 1 is the fourth Part of 4.) you may more readily take the fourth Part of 86 Feet 4 Inches 8 Parts, which is also 21 Feet 7 Inches 2 Parts; then 2 Inches are the sixth of 12, take the sixth of 48 Inches will be 8 Inches, which place under the Inches; then add all together, and the Sum is 12677 Feet 6 Inches 10 Parts. See the foregoing Work.

I shall set down only the Working of some few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, &c.

	F.	I.		F.	I.
	179	3		246	7
	38	10		146	4
	<hr/>			<hr/>	
	1432			1476	
	537	P.		984	P.
	89	7 6		82	2 4
	59	9 0		15	4 0
	9	6 0		11	6 0
	<hr/>			<hr/>	
Product	6960	10 6	Product	11425	0 4
	F.	I.		F.	I.
	246	7		257	9
	36	9		39	11
	<hr/>			<hr/>	
	1476			2313	
	738	P.		771	P.
	123	3 6		128	10 6
	61	7 9		85	11 0
	12	0 0		21	5 9
	9	0 0		19	6 0
	<hr/>			9	9 0
Product	9061	11 3	Product	10288	6 3
					<i>Example</i>

Chap. 9. Multiplication of Feet, &c. 65

Example 11. Let 7 Feet 5 Inches 9 Parts be multiply'd by 3 Feet 5 Inches 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
<hr/>				
22	5	3	S	
3	1	4	9	T.
	1	10	5	3
<hr/>				
25	8	6	2	3

In this Example, I first begin with 3 Feet, and thereby multiply 7 Feet 5 Inches 9 Parts: First, I say, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; set down 3 under the Parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; set down 5 Inches, and carry 1, and say, 3 times 7 is 21, and 1 I carry is 22; set down 22 Feet. Then begin with 5 Inches, saying, 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; set down 9 Seconds a Place towards the Right-hand, and carry 3 Parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; set down 4 Parts, and carry 2, saying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; set down 3 Feet 1 Inch, and begin to multiply by 3 Parts, saying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; set down 3 Thirds, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; set down 5 Seconds, and carry 1, saying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which set down, and add all up, and the Product is 25 Feet 8 Inches 6 Parts 2 Seconds 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, &c. if Feet be multiply'd by Feet, the Product is Feet; and Feet multiply'd by Inches, the Product is Inches;

66 *Multiplication of Feet, &c. Part I.*

and the twelfth Part is Feet ; and Parts multiply'd by Feet, the Product is Parts, and the twelfth Part thereof is Inches ; Parts multiply'd by Inches, the Product is Seconds, and the twelfth Part thereof is Parts ; and Parts multiply'd by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the first Row, and Parts by Inches in the second Row, and Parts by Parts in the third Row, the first Figure in every Row will stand a Place more towards the Right-hand, as you may see in the last Example.

Example 12. Let 37 Feet 7 Inches 5 Parts be multiply'd by 4 Feet 8 Inches 6 Parts.

F. I. P.			
37	7	5	
4	8	6	
<hr/>			
150	5	8	S
12	6	5	8
12	6	5	8 T
1	6	9	8 6
<hr/>			
177	1	5	0 6

First, I multiply by 4 Feet, saying, 4 times 5 is 20, which is 1 Inch 8 Parts ; set down 8, and carry 1, saying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet 5 Inches ; set down 5 Inches, and carry 2, saying, 4 times 7 is 28, and 2 I carry is 30 ; set down 0, and carry 3, and say, 4 times 3 is 12, and 3 is 15 ; set down 15. Then I begin with 8 Inches ; but, because the Feet in the Multiplicand are more than 12, it will be the best Way to work for the aliquot Parts of 8 ; so here I work for 4 Inches, and set that down twice, 4 being the third Part of 12 ; therefore take the third Part of 37 Feet 7 Inches 5 Parts, which is twelve Feet six Inches five Parts eight Seconds ; set this down twice. Then begin with 6 Parts ; but, instead of multiplying, take half 37 Feet 7 Inches 5 Parts,

Chap. 9. *Multiplication of Feet, &c.* 67

5 Parts, (because 6 is half 12) and set it a Place more to the Right-hand: Thus, the half of 37 Feet is 18, which I must count 18 Inches, because the Multiplier is 6 Parts; so the half of 37 Feet 7 Inches 5 Parts, is 1 Foot 6 Inches 9 Parts 8 Seconds 6 Thirds; which set down, and add all up together, and the Sum is 177 Feet 1 Inch 5 Parts 0 Seconds 6 Thirds.

Example 13. Let 311 Feet 4 Inches 7 Parts be multiply'd by 36 Feet 7 Inches 5 Parts.

F. I. P.					
311	4	7			
36	7	5			
<hr/>			<hr/>		
1866					
933			S		
103	9	6	4		
77	10	1	9	T	
8	7	9	6	4	
2	1	11	4	7	
12	0	0	0	0	
1	0	0	0	0	
	9	0	0	0	
<hr/>			<hr/>		
11402	2	4	11	11	

In this Example, because the Feet both in the Multiplier and Multiplicand are compound Numbers, I first multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; so take the third Part of 311 Feet 4 Inches 7 Parts, which is 103 F. 9 I. 6 P. 4 S. and the fourth Part is 77 F. 10 I. 1 P. 9 Sec set these down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; so the third Part of 311 Feet 4 Inches 7 Parts is 103 F. 9 I. 6 P. 4 Sec. but because the Multiplier is Parts, it must be set a Place to the Right-hand, that

that is, the 103 must be Inches, which is 8 Feet 7 Inches; therefore I set down 8 F. 7 I 9 P. 6 Sec. 4 Th. Then, because 1 Inch is a fourth Part or 4 Inches, therefore I take a fourth Part of 8 F. 7 I. 9 P. 6 Sec. 4 Th. which is 2 F. 1 I. 11 P. 4 Sec. 7 Th. which is the same as if I had taken a twelfth Part of 311 F. 4 I. 7 P. Then for 4 Inches in the Multiplicand, instead of Multiplying 36 Feet by it, take a third Part, because 4 Inches is the third Part of 12; so the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a third and a fourth; so the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all these together, and the Sum will be 11402 Feet 2 Inches 4 Parts 11 Seconds 11 Thirde.

Example 14. Let 8 Feet 4 Inches 3 Parts 5 Seconds 6 Thirds be multiply'd by 3 Feet 3 Inches 7 Parts 8 Seconds 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	<hr/>								
25	0	10	4	6					
2		0	10	4	6				
	1	10	6	0	2	6			
		5	6	10	3	8	0		
			1	4	8	6	11	0	
	<hr/>								
Prod.	27	7	3	5	1	8	8	11	0

In this last Example there is no Difficulty, if you do but observe the former Directions, and set every Row a Place more to the Right Hand.

Chap. 9. *Multiplication of Feet, &c.* 69

I shall only set down the Working of some few Examples more, and so conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
<hr/>				
2894	5	3	S	
80	4	9	9	T
13	4	9	7	6
<hr/>				
2988	2	10	4	6
<hr/>				

F.	I.	P.		
42	7	8		
7	3	6		
<hr/>				
298	5	8		
10	7	11	S	
1	9	3	10	
<hr/>				
310	10	10	10	
<hr/>				

F.	I.	P.		
124	7	9		
14	6	2		
<hr/>				
496				
124			S	
62	3	10	6	T
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	2	6	0	0
<hr/>				
1809	1	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
<hr/>				
2072				
259			S	
86	7	6	8	
21	7	10	8	T
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
<hr/>				
4793	6	0	10	8

70 *Multiplication of Feet, &c. Part I.*

F.	I.	P.
267	7	10
25	9	7

1335				
534				
133	9	11	8	
66	10	11	6	
11	1	9	11	T
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	1	0	0

6905	0	7	0	10
------	---	---	---	----

F.	I.	P.
317	9	7
37	5	9

2219				
951				S
105	11	2	4	
26	5	9	7	T
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0

11910	9	11	1	3
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T H E



THE
Complete Measurer, &c.

PART II.

CHAP. I.

The Mensuration of SUPERFICIES.

Superficial Figures are all such as have only Length and Breadth, not having any commensurable Thickness.

§ I. Of a SQUARE.

A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or square) Angles. To find the superficial Content thereof, this is

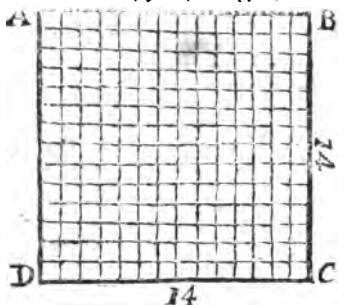
The RULE.

Multiply the Side into itself, and the Product is the Content.

Let

72 *Mensuration of Superficies. Part II.*

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Measure; multiply 14 by itself, and the Product is 196, which is the superficial Content.



$$\begin{array}{r}
 14 \\
 14 \\
 \hline
 56 \\
 14 \\
 \hline
 196 \text{ Product.}
 \end{array}$$

By Scale and Compasses.

Extend the Compasses from 1, in the Line of Numbers, to 14; the same Extent will reach from the same Point, turn'd forward to 196.

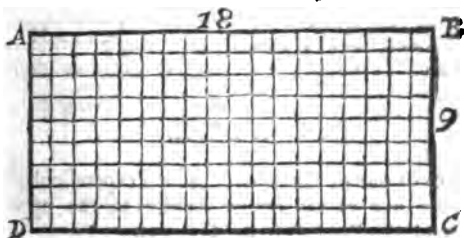
Demonstration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, crossing each other within the Square; so shall the whole great Square be divided into 196 little Squares, as you may see in the Figure above, equal to the Number of Square Feet, Yards, or Poles, or other Measures, by which the Side was measur'd.

§ II. Of a PARALLELOGRAM, or
Long Square.

A Parallelogram is a Figure having four Sides, and as many Right Angles, the opposite Sides thereof being equal and parallel. To find the superficial Content thereof, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the superficial Content.



$$\begin{array}{r} \text{Length } 18 \\ \text{Breadth } 9 \\ \hline \text{Product } 162 \end{array}$$

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet; which multiply'd together, the Product is 162, the superficial Content thereof.

By Scale and Compaffes.

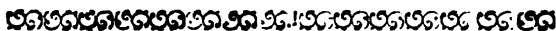
Extend the Compaffes in the Line of Numbers from 1 to 9, the fame Extent will reach from 18 down to 162, the fquare Feet.

H

Demon-

74 *Mensuration of Superficies. Part II.*

Demonstration. If the Sides A B and C D be each divided into 18 equal Parts, representing 18 Feet; and the Lines A D and B C each divided into 9 equal Parts, and Lines drawn from Point to Point, crossing each other within the Figure; those Lines will make thereby so many little Squares as there are square Feet, viz. 162.



§ III. Of a R H O M B U S.

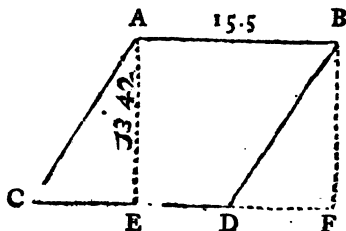
A Rhombus is a Figure representing a Quarry of Glas, having four equal Sides, the Opposites thereof being equal, two Angles being obtuse, and two acute. To find the superficial Content thereof, this is.

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuse Angles to the opposite Side, and the Product is the Content.

Perpend.	13.42
The Side	15.5
	<hr/>
	6710
	6710
	1342
	<hr/>

Product 208.010



Let ABCD be a Rhombus given, whose Sides are each 15.5 Feet, and the Perpendicular E A is 13.42, which multiply'd together, the Product is 208.010; which is the superficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By Scale and Compasses.

Extend the Compasses from 1 to 13.42. that Extent will reach from 15.5, the same Way to 208 Feet the Content.

Demonstration. Let CD be extended out to F, making DF equal to CE, and draw the Line BF; so shall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, because AB and CF are parallel. Therefore the Parallelogram ABEF is equal to the Rhombus ABCD.



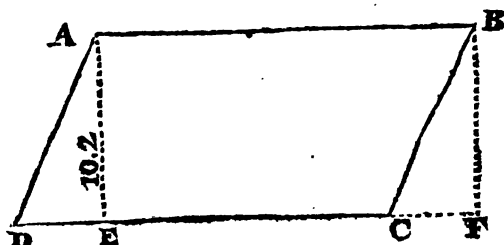
§ IV. *Of a R H O M B O I D E S.*

A Rhomboides is a Figure having four Sides, the opposite whereof are equal and parallel; and also four Angles, the opposite whereof are equal. To find the superficial Content thereof, this is

The R U L E.

Multiply one of the longest Sides thereof by the Perpendicular let fall from one of the obtuse Angles to one of the longest Sides, and the Product is the Content.

$$\begin{array}{r}
 19.5 \\
 10.2 \\
 \hline
 390 \\
 195 \\
 \hline
 198.90
 \end{array}$$



Let ABCD be a Rhomboides given, whose longest Sides, AB or CD, is 19.5 Feet, and the Perpendicular AE is 10.2 ; which multiply'd together, the Product is 198.9, that is, 198 superficial Feet and 9 tenth Parts, the Content.

Demonstration. If DC be extended to F, making CF equal to DE, and a Line drawn from B to F ; so will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD ; which was to be proved.



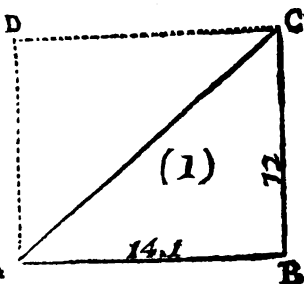
§ V. Of a TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are such as have one Right Angle. Oblique-angled Triangles are such as have their Angles either acute or obtuse. An obtuse Angle is greater than a Right Angle, that is, it is more than 90 Degrees ; and an acute Angle is less than a Right Angle. To find the superficial Content thereof, this is

The R U L E.

Let the Triangle be of what kind soever, multiply the Base by half the Perpendicular, or half the Base by the whole Perpendicular; or, multiply the whole Base by the whole Perpendicular, and take half the Product; any of these three Ways will give the Content.

Let ABC be a Right - angled Triangle, whose Base is 14.1 Feet, and the Perpendicular 12 F. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or, multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the same as before.



14.1 Base.

6 Half Perpendicular.

84.6 Product.

14.1 Base.

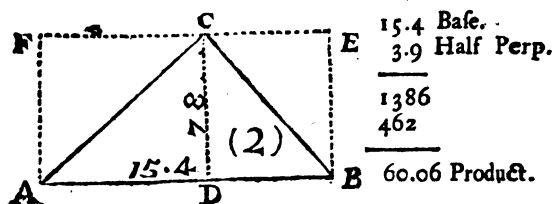
12 Perpendicular.

169.2 Product.

84.6 Half.

By Scale and Compasses.

Extend the Compasses from 2 to 14.1, that Extent will reach the same Way from 12 to 84.6 Feet, the Content.



15.4 Base.	7.7 Half Base.
7.8 Perpend.	7.8 Perpendicular.
<hr/>	<hr/>
1232	616
1078	539
<hr/>	<hr/>
120.12	60.06 Product.
<hr/>	
60.06 Half.	

Let ABC (Fig. 2.) be an oblique-angled Triangle given, whose Base is 15.4, and the Perpendicular 7.8; if 15.4 be multiply'd by 3.9 (half the Perpendicular), the Product will be 60.06 for the Area or superficial Content: Or, if the Perpendicular 7.8 be multiply'd into half the Base 7.7, the Product will be 60.06 as before: Or, if 15.4, the Base, be multiply'd by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the Half thereof is 60.06 Feet, as before. See the Work.

By Scale and Compasses.

Extend the Compasses from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

Demonſtration. If AD (Fig. 1.) be drawn parallel to BC , and DC parallel to AB ; the Triangle ADC shall be equal to the given Triangle ABC . Hence the Parallelogram $ABCD$ is double to the given Triangle; therefore half the Area of the Parallelogram

Chap. 1. Mensuration of Superficies. 79

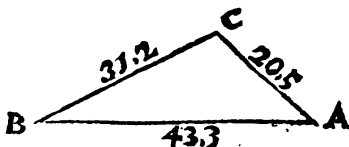
is the Area of the Triangle. In Fig. 2. the Parallelogram ABEF is also double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be prov'd.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum: then subtract each Side severally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the last Product extract the Square Root; which square Root shall be the Area of the Triangle sought.

Example. Let ABC be a Triangle, whose three Sides are as followeth, viz. AB 43.3, AC 20.5, and BC 31.2, the Area is requir'd.



$$\text{Sides } \left\{ \begin{array}{l|l} 43.3 & 4.2 \\ 31.2 & 16.3 \\ 20.5 & 27.0 \end{array} \right\} \text{Differences.}$$

$$\text{Sum } 95.0$$

$$\text{Half } 47.5$$

Area

80 Mensuration of Superficies. Part II.

Area 296.31 47.5 The half Sum.
27 Difference.

3325
950

1282.5 Product.
16.3 Difference.

38475
76950
12825

20904.75 Product.
4.2 Difference.

4180950
8361900

87799.9500(296.31

4

49)477
441

586)3699
3516

5923)18395
17769

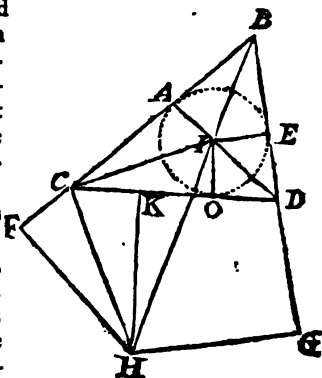
59261)62600
59261

3339 Remains.

Chap. I. *Mensuration of Superficies.* 81

Demonstration. In the Triangle BCD, I say, if from the half Sum of the Sides, you subtract each particular Side, and multiply the half Sum and the three Differences together continually, the square Root of the Product shall be the Area of the Triangle.

First, by the Lines BI, CI, and DI, bisect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new



Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI, and OI, being all equal to one another, because the Point I is the Centre of the inscribed Circle (by *Euclid*, Lib. 4. *Prop.* 4.): Wherefore to the Side BC join CF equal to DE, or DO; so shall BF be equal to half the Sum of the Sides, viz. $= \frac{1}{2} BC + \frac{1}{2} BD + \frac{1}{2} CD$.

And $BA = BF - CD$; for $CA = CO$ and $OD = CF$; therefore $CD = AF$; and $AC = BF - BD$, for $BE = BA$, and $ED = CF$; therefore $BD = BA + CF$, and $CF = BF - BC$.

Then make $CK = CF$, and draw the Perpendiculars FH, GH, and KH, and extend BI to H; because the Angles FCK + FHK are equal to two right Angles (for the Angles F and K are right Angles), equal also to FCK + ACO (by *Euclid*. 1. 13.) and the Angles ACO + AIO are equal to two right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are also similar. And the Triangles BAI and BFH are likewise similar.

From

82 *Mensuration of Superficies. Part II.*

From this Explanation, I say, the Square of the Area of the given Triangle will be $BF \text{ } q \times IA \text{ } q = BF \times BA \times CA \times CF$. In Words;

The Square of BF (the half Sum of the Sides) multiply'd into the Square of IA (=IF=IO) will be equal to the said half Sum multiply'd into all the three Differences.

For $IA : BA :: FH : BF$; and $IA : CF :: AC : FH$; because the Triangles are similar. *By Euclid. Lib. 6. Prop. 4.*

Wherefore multiplying the Extremes and Means in both, it will be $IA \text{ } q \times BF \times FH = BA \times CA \times CF \times FH$; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be $BF \text{ } q \times IA \text{ } q = BF \times BA \times CA \times CF$. Which was to be demonstrated.



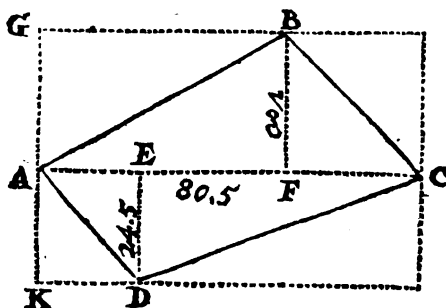
§ VI. *Of a TRAPEZIUM.*

A Trapezium is a Figure having 4 unequal Sides, and oblique Angles. To find the Area or superficial Content thereof, this is

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Area's of the two Triangles, ABC and ACD, (by Section V.) and add those two Area's together, the Sum shall be the Area of the Trapezium.

$$BF=30.1$$



$$BF = 30.1$$

$$DE = 24.5$$

$$\text{Sum } 54.6$$

$$\text{Half } 27.3$$

$$AC = 80.5$$

$$1365$$

$$2184$$

$$\text{Area } 2197.65$$

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; these two added together, the Sum is 54.6, the Half thereof is 27.3, which multiply'd by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiply'd by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the same as before.

By

84: *Multiplication of Superficies. Part II.*

By Scale and Compasses.

Extend the Compasses from 2 to 54.6; that Extent will reach from 80.5 to 21976.65, the Area.

Demonstration. This Figure ABCD is compos'd of two Triangles; the Triangle ABC is half the Parallelogram AGHC: Also the Triangle ACD is equal to half the Parallelogram ACIK, as was prov'd Sect. V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area $HI=BF+DE$; therefore $\frac{1}{2} HI \times AC (=KI=GH) = \text{Area of the Trapezium.}$ Which was to be prov'd.



§ VII. Of Irregular FIGURES.

Irregular Figures are all such as have more Sides than four, and the Sides and Angles unequal. All such Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of such Figures, they must be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another; and so find the Area's of the Trapeziums and Triangles severally, and then add all the Area's together; so will you have the Area of the whole Figure.

Let ABCDEFG be an irregular Figure given to be measur'd; first, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Area's severally.

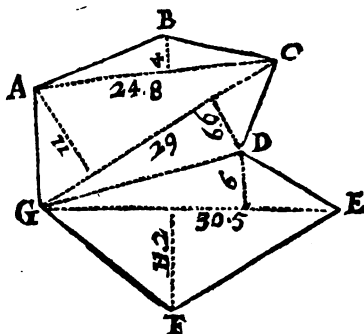
First, I multiply the Base AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then

Chap. 1. *Mensuration of Superficies.* 85

Then for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiply'd by 29, the Diagonal; the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the Half thereof is 8.6; which multiply'd by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All these Area's added together, make 567.1, and so much is the Area of the whole irregular Figure. See the Work.



24.8 Base AC.
2 Half Perpendicular.

49.6 Area of ABC.

11 Perpendiculars.
6.6

17.6 Sum.
8.8 Half.
29 Diag. CG.

792
176

255.2 Area of ACGD.

86 *Mensuration of Superficies.* Part II.

11.2	} Perpendiculars.	30.5	
6		8.6	
<hr/>		<hr/>	
17.2	Sum,	1830	
<hr/>		<hr/>	
8.6	Half Sum.	2440	
		<hr/>	
		262.30	Area of GDEF:
		255.2	Area of ACGD;
		49.6	Area of ABC.
		<hr/>	
		567.1	Sum of the Areas.

This Figure being compos'd of Triangles and Trapeziums, and those Figures being sufficiently demonstrated in the Vth. and VIth. Sections. aforegoing, it will be needless to mention any thing of the Demonstration thereof in this Place.



§ VIII. Of Regular POLYGONS.

Regular Polygons are all such Figures as have more than four Sides, all the Sides and Angles thereof being equal. Polygons are denominated from the Number of their Sides and Angles.

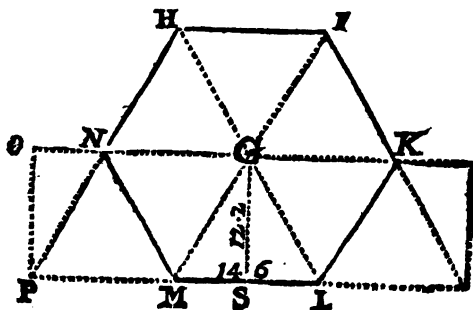
If the Figure consists of	5	Equal Sides and Angles, it is called a regular	Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
	9		Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

Chap. I. *Mensuration of Superficies.* 87

To find the Area or superficial Content of any regular Polygon, this is

The R U L E.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular let fall from the Centre to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



$$\begin{array}{r}
 14.6 \\
 3 \\
 \hline
 43.8 \text{ Half Sum of the Sides.} \\
 12.64 \text{ The Perpendicular.} \\
 43.8 \text{ Half Sum.} \\
 \hline
 10112 \\
 3792 \\
 \hline
 5056 \\
 \hline
 553.632 \text{ Area.}
 \end{array}$$

$$\begin{array}{r}
 14.6 \\
 6 \\
 \hline
 87.6 \text{ Sum of the Sides.} \\
 6.32 \text{ Half Perpend.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1752 \\
 2628 \\
 \hline
 5256 \\
 \hline
 553.632 \text{ Area.}
 \end{array}$$

88 *Mensuration of Superficies. Part II.*

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiply'd by the Perpendicular GS 12.64, the Product is 553.63: Or, if 87.6, the whole Sum of the Sides, be multiply'd by half the Perpendicular 6.32, the Product is 553.63, the same as before, which is the Area of the given Hexagon.

By Scale and Compasses.

Extend the Compasses from 1 to 12.2, that Extent will reach from 43.8, the same Way to 553.63. Or, extend from 2 to 12.2, that Extent will reach from 87.6 to 553.63.

Demonstration. Every regular Polygon is equal to the Parallelogram, or long Square, whose Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of six equilateral Triangles: And the Parallelogram OPQR is also compos'd of six equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

A T A B L E

A TABLE for the more ready finding
the Area of a Polygon.

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

How to find these tabular Numbers.

These Numbers are found by Trigonometry, thus: Find the Angle at the Centre of the Polygon by dividing 360 Degrees by the Number of Sides of the Polygon.

Example. Suppose each Side of the Dodecagon annexed by 1, and the Area be required.



90 *Mensuration of Superficies.* Part II.

Divide 360 by 12 (the Number of Sides), and the Quotient is 30 Degrees for the Angle ACB; the Half thereof is 15, the Angle DCB, whose Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then say,

As s, DCB 15 Degrees is	Co-ar. 0.587004
to .5 the Half-side DB. Log.	1.698970
so is s, CBD 75 Degrees.	9.984944
	<hr/>
to the Perpendicular CD 1.866025	0.270918
	<hr/>

Then 1.866025 multiply'd by .5 (the Half side), the Product is 9.330125 the Area of the Dodecagon requir'd.



§ IX. Of a CIRCLE.

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure, called the Centre, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compass.

Problem 1. Having the Diameter and Circumference, to find the Area.

The RULE.

Every Circle is equal to a Parallelogram, whose Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply

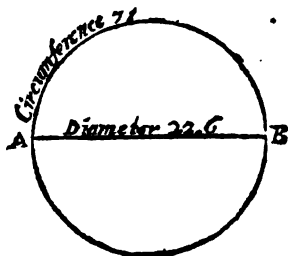
Chap. I. *Mensuration of Superficies.* 91

ply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Cirumf.
11.3 Half Diameter.

1065
= 355
355

401.15 Area.



Thus, if the Diameter of a Circle (that is, the Line drawn cross the Circle through the Centre) be 22.6; and if the Circumference be 71, the Half of 71 is 35.5, and the Half of 22.6 is 11.3; which multiply'd together, the Product is 401.15, which is the Area of the Circle.

Demonstration. Every Circle may be conceiv'd to be a Polygon of an infinite Number of Sides, and the Semidiameter must be equal to the Perpendicular of such a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiply'd by half the Diameter, gives the Area as aforesaid.

Or (with *F. Ignat. Gaston Pardies*), "Every Circle is equal to a Rectangle-Triangle, one of whose Legs is the Radius, and the other a right Line equal to the Circumference of the Circle: For such a Triangle will be greater than any Polygon inscrib'd, and less than any Polygon circumscrib'd by the 24th, 25th, 26th, and 27th Articles of the fourth Book of his Elements of Geometry); and therefore must be equal to the Circle.

" For

92 *Mensuration of Superficies. Part II.*

“ For (says he) should it be greater than the
 “ Circle, be the Excess as little as it will, a Polygon
 “ may be circumscrib'd, whose Difference, from the
 “ Circle, shall be yet less than the Difference between
 “ that Circle and the Rectangle-Triangle; and that
 “ that Polygon will be less than the Triangle, is ab-
 “ surd; and if it be said, that this rectangled Triangle
 “ is less than the Circle, an inscrib'd Polygon may be
 “ made, which shall be greater than that Triangle;
 “ which is impossible.

“ This cannot but be admitted as a Principle,
 “ That if two determinate Quantities, A and B,
 “ are such that if every imaginable Quantity,
 “ which is greater or less than A, is also greater or
 “ less than B, these two Quantities A and B must be
 “ equal.

“ And this Principle being granted, which is in
 “ a manner self-evident, it may directly be prov'd,
 “ that the Triangle (before mentioned) is equal to
 “ the Circle; because every imaginable inscrib'd
 “ Figure, which is less than the Circle, is also less
 “ than the Triangle; and every circumscrib'd Figure,
 “ greater than the Circle, is also greater than the
 “ Triangle.”

Problem 2. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, so is the Diameter to the Circumference.

Or, as 113 to 355, so is the Diameter to the Circumference.

Or, as 1 to 3.141593, so is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiply'd by 22, and the Product is 497.2; which, divided by 7, gives 71.028 for the Circumference.

Chap. I. *Mensuration of Superficies.* 93

ference. Or (by the second Proportion) if 22.6 be multiply'd by 355; the Product will be 8023; this divided by 11.3, the Quotient is 71, the Circumference. Or, (by the third Proportion) if 22.6 be multiply'd into 3.141593, the Product is 71.0000018 the Circumference; which two last Proportions are the most exact.

$ \begin{array}{r} 22.6 \\ 22 \\ \hline 452 \\ 452 \\ \hline 71497.2(71.028 \end{array} $	$ \begin{array}{r} 355 \\ 22.6 \\ \hline 2130 \\ 710 \\ 710 \\ \hline 113)8023.01(71 \\ 791 \\ \hline 113 \\ 113 \\ \hline \dots \end{array} $
$ \begin{array}{r} 3.141593 \\ 22.6 \\ \hline 18849558 \\ 6283186 \\ 6283186 \\ \hline 71.0000018 \end{array} $	

By Scale and Compasses.

Extend the Compasses from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle to the Circumference was never yet exactly found, notwithstanding many eminent learned Men have labour'd very far therein; among which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the said Proportion to 36 Places of Decimals, which are engraven upon his Tomb-stone in *St. Peter's Church* in *Leyden*; which Numbers are these;

Diameter,

94 *Mensuration of Superficies. Part II.*

Diameter.

1.00000.00000.00000.00000.00000.00000.00000

Circumference.

3.14159.26535.89793.23846.26433.83279.50288

Of which large Number these six Places, 3.14159, answering to the Diameter 1.00000, may be sufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I shall leave my Reader to use which of them he pleases, but shall commend the last two as most exact, tho' the first be most in Use; but in the following Work I shall use sometimes one of them, and sometimes another, but for the most Part that of *Van Culen*, as being most exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309, so is the Circumference to the Diameter.

Or, as 355 to 113, so is the Circumference to the Diameter.

Or, as 22 to 7, so is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiply'd by 71, (as by the first Proportion) the Product will be 22.599939 for the Diameter. Or, by the second Proportion, 113 multiply'd by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiply'd by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309	113	71
71	71	7
<hr/>	<hr/>	<hr/>
318309	113	22)497(22.59
2228163	791	44
<hr/>	<hr/>	<hr/>
22.599939	355)8023(22.6	57
	710	44
	<hr/>	<hr/>
	923	130
	710	110
	<hr/>	<hr/>
	2130	200
	2130	198
	<hr/>	<hr/>
	...	2

Thus, by both the first Proportions, the Diameter is 22.6, but by the last it falls something short.

By Scale and Compasses.

Extend the Compasses from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter sought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the same will reach from 71 to 22.6, as before.

Note, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters (by *Euc.* 12.2). Now, the Area of a Circle, whose Diameter is 1, will be .785398, according to *Van Gulen's* Proportion before

96 *Mensuration of Superficies. Part II.*

fore-mention'd; but for Practice .7854 will be sufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, so is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.15 (the Area of the given Circle): But,

According to *Metius's* Proportion,

As 452 : 355 :: 510.76 : 401.15, the same as before.

But, if you use *Archimedes's* Proportion, say,

As 14 : 11 :: 510.76 : 401.31; which Area is greater than by the two former Proportions; though in small Circles this is near enough the Truth. See the Working of all these.

22.6 Diameter of the former Circle.

22.6

1356

452

452

510.76 The Square of the said Diameter.

As 1 : .7854 :: 51076

.7854

204304

255380

408608

357532

401.150904 the Area.

By

Chap. I. Mensuration of Superficies. 97

By Scale and Compasses.

The Extent from 1 to 22.6, being twice turn'd over from .7854, will fall at the last upon 401.15, the Area.

$$\begin{array}{r} 113 \\ 4 \\ \hline 452 \end{array}$$

$$\text{As } 452 : 355 :: 510.76$$

$$\begin{array}{r} 355 \\ \hline 255380 \\ 255380 \\ \hline 153228 \end{array}$$

$$\begin{array}{r} 452 \quad 181319.80(401.15 \\ \hline 1808 \end{array}$$

$$\begin{array}{r} 519 \\ 452 \\ \hline \end{array}$$

$$\begin{array}{r} 678 \\ 452 \\ \hline \end{array}$$

$$\begin{array}{r} 2260 \\ 2260 \\ \hline \end{array}$$

...

K

As

98 *Mensuration of Superficies. Part II.*

As 14 : 11 :: 510.76

$$\begin{array}{r}
 11 \\
 \hline
 14 \overline{) 510.76} \quad 36 \quad 401.31 \\
 \underline{56} \\
 18 \\
 \underline{14} \\
 43 \\
 \underline{42} \\
 16 \\
 \underline{14} \\
 2
 \end{array}$$

Problem 5. Having the Circumference of a Circle, to find the Area.

Because the Diameters of Circles are proportional to their Circumferences ; that is, as the Diameter of one Circle is to its Circumference ; so is the Diameter of another Circle to its Circumference : Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958 ; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

	Sq. Cir.	Area.	Sq. Circumf.
As	1 :	.07958 ::	5041
			<u>5041</u>
			7958
22			31832
<u>4</u>			<u>397900</u>
88			401.16278

Or

Or thus :

$$\begin{array}{r}
 \text{As } 88 : 7 :: 5041 \\
 \hline
 7 \\
 \hline
 88 \overline{) 35287} (400 \text{ 98 Area} \\
 \underline{352} \\
 870 \\
 \underline{792} \\
 780 \\
 \underline{704} \\
 76 \\
 \hline
 355 \\
 \underline{4} \\
 1420
 \end{array}$$

Or, As 1420 : 113 :: 5041 : 401.15 Area

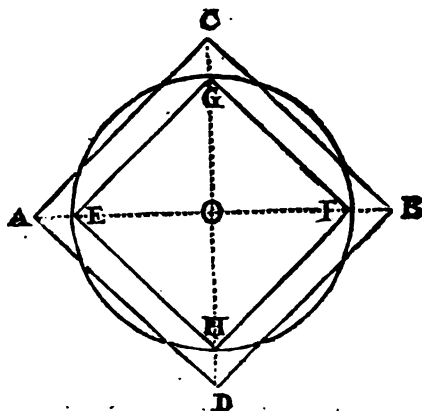
Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

As 1 : .8862 :: 22.6 (the Diameter)

$$\begin{array}{r}
 22.6 \\
 \hline
 53172 \\
 17724 \\
 \hline
 17724
 \end{array}$$

To 20.02812 the Side of the Square A C.



Let the Diameter of the Circle EF or GH, be 22.6 (as before), to find the Side of the Square AC, AD, &c. If .8862 be multiply'd by 22.6 the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiply'd Square-wise, that is, by itself, it will produce 401.1255907344. which is nearly equal to the Area found in the last Problem.

You may find the Side of the Square equal, by extracting the Square Root out of the Area of the given Circle.

Chap. I. Mensuration of Superficies. 101

401.15 (20.0237295 Side of the Square.

4

4002)01.1500

8004

40048)349600

.... 320384

29216

28034

1182

801

381

360

21

20

1

N.B. By this Method of extracting the squareRoot of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

As 1 : .2821 :: 71 (the Circumference).

71

2821

19747

20.0291 the Side of the Square.

K 3

Problem

102 *Mensuration of Superficies.* Part II.

Problem 8. Having the Diameter, to find the Side of a Square, which may be inscrib'd in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inscrib'd will be .7071. Therefore,

As 1 : .7071 :: 22.6.

22.6

42426

14142

14142

To 15.98046 the Side E G inscrib'd.

Or, if you square the Semidiameter, and double that Square, the square Root of the doubled Square will be the Side of the Square inscrib'd. For (by *Euclid*. 1.47) the Square of the Hypothenuse E.G. is equal to the Sum of the other two Legs, E O and O G.

11.3 Semidiameter.

11.3

339

113

113

127.69 the Squ. of EO, which double, because
2 (EO=OG.

255.38 (15.98 Root, which is the Side of the Sq.

25)155

125

309)3038

2781

3188)25700

25504

196

Problem

Chap. 1. *Mensuration of Superficies.* 103

Problem 9. Having the Circumference, to find the Side of a Square which may be inscrib'd.

If the Circumference be 1, the Side of the Square inscrib'd will be .2251. Therefore,

$$\text{As } 1 :: .2251 :: 71$$

71

2251

15757

15.9821 the Side of the Squ. EG.

Because that in each of the four last Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be easy to work them with Scale and Compasses; for if you extend the Compasses from the first to the second, that Extent will reach from the third to the fourth: As in the last Problem, where the Proportion is as 1 to .2251, so is 71 to the Side of the Square 15.9821. Here extend the Compasses from 1 to .2251; that Extent will reach from 71 to 15.98; and so of the rest. But the fifth must be wrought like the 4th, thus; extend the Compasses from 1 to 71; that Extent turn'd over the same way from .07958, will fall, at the last, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

104 *Menfuration of Superficies. Part II.*

Area. Sq. Diam. Area.

As 1 : 1.2732 :: 401.15.

401.15

63660

12732

12732

509280

510.744180 (22.599 the Diameter.

4

42)110

84

445)2674

2225

4509)44941

40581

45189)436080

406701

29379

By Scale and Compaffes.

Extend the Compaffes from 1 to 1.2732 ; that Extent will reach from 401.15 to 510.74, &c. Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and .7854, into two equal Parts, and one of thofe Parts will reach from 1 to 22.6, the Diameter fought.

Problem

Chap. I. Mensuration of Superficies. 105

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

Ar. Sq. Circumf. Area.

As 1 : 12.56637 :: 401.15

401.15

6283185

1256637

1256637

50265480

. Circumf.

5040.99932550(70.9990 Root.

49

1409)14099

12681

14189)141893

127701

141989)1419225

1277901

1419989)14132450

12779901

1352549

By Scale and Compasses.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of those Parts will reach from 1 to 71, the Circumference sought.

Problem

106 *Mensuration of Superficies. Part II.*

Problem 12. Having the Area, to find the Side of a Square inscrib'd.

If the Area of a Circle be 1, the Area of a Square inscrib'd within that Circle will be .6366. Therefore,

$$\text{As } 1 : 401.15 :: .6366$$

$$\begin{array}{r} \text{.6366} \\ \hline 240690 \\ 240690 \\ 120345 \\ 240690 \\ \hline \end{array}$$

255.372090 (15.98 Root, which is the Side of the
1 (Square sought.)

$$\begin{array}{r} 25)155 \\ 125 \\ \hline \end{array}$$

$$\begin{array}{r} 309)3037 \\ 2781 \\ \hline \end{array}$$

$$\begin{array}{r} 3188)25620 \\ 25504 \\ \hline \end{array}$$

$$11690$$

The same Reason may be given for the last Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but also all Figures inscribed or circumscrib'd, have the Squares of their like Sides proportional to the Circles they are inscrib'd in, or circumscrib'd about; and also to the Figures themselves: The Square of any Side of one Figure is to the Area of that

Chap. I. *Mensuration of Superficies.* 107

that Figure, as the Square of the like Side of another similar Figure is to the Area thereof: as you may find prov'd at large in *Euclid*, *Sturmius*, *Matheſis Enucleata*, and other Authors; but will be too large to insert in this Place.

By Scale and Compasses.

Extend the Compasses from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumscribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumscribe that Square, will be 1.4142. Therefore,

As 1 : 1.4142 :: 15.98

15.98,

113136

127278

70710

14142

22.598916 the Diameter sought.

By Scale and Compasses.

Extend the Compasses from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter sought.

Problem

108 *Mensuration of Superficies. Part II.*

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,
Side Diam. Side of a Square.

As 2 : 1.128 :: 20.0291.

1.128

1602328

400582

200291

200291

22.5928248 Diameter.

By Scale and Compasses.

Extend the Compasses from 1 to 1.128; that Extent will reach from 20 0291, (the Side of the Square given) to 22.6, the Diameter of the Circle sought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumscribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompass that Square will be 4.443. Therefore,

Side Sq. Circum. Side Sq.

As 1 : 4.443 :: 15.98

15.98

35544

39987

22215

4443

70.99914 the Circumf.

By Scale and Compasses.

Extend the Compasses from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem

Chap. I. *Mensuration of Superficies.* 109

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto, shall be 3.545. Then,

As 1 : 3.545 :: 20.0291

$$\begin{array}{r}
 3.545 \\
 \hline
 1001455 \\
 801164 \\
 1001455 \\
 600873 \\
 \hline
 \end{array}$$

71.0031595 the Circumference.

By Scale and Compasses.

Extend the Compasses from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference sought.

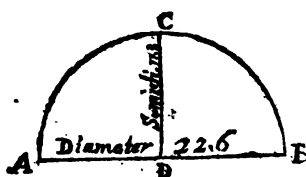
In several of the foregoing Problems, where the Diameter and Circumference is requir'd, the Answers are not exactly the same as the Diameter and Circumference of the given Circle, but are sometimes too much, and sometimes too little, as in the two last Problems, where the Answers in each should be 71, the one being too much, and the other too little. The Reason of this is, the small Defect that happens to be in the Decimal Fractions, they being sometimes too great, and sometimes too little; yet the Defect is so small, that it is needless to calculate them to more Exactness.

§ X. Of a SEMICIRCLE.

TO find the Area of the Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whose Diameter 22.6, and the half Circumference, or Arch-line, ACB, is 35.5. the Half thereof is 17.75. which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

17.75 the half Arch-line.
11.3 the Semidiameter.

$$\begin{array}{r} 1775 \\ 1775 \\ \hline 5325 \\ 1775 \\ \hline 200575 \end{array}$$

200.575 the Area of the Semicircle.

By Scale and Compasses.

Extend the Compasses from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area!

If only the Diameter of the Semicircle be given, you may say, by the *Rule of Three*;

As 1 is to .3927, so is the Square of the Diameter to the Area.

Bj

By Scale and Compasses.

Extend the Compasses from 1 to the Diameter 22.6; that Extent turn'd twice from .3927, will reach, at the last, to 200.575.

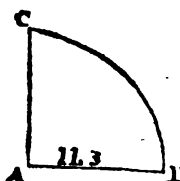


§ XI. *Of a Q U A D R A N T.*

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The R U L E.

Multiply half the Arch-line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle), by the Semidiameter, and the Product is the Area of the Quadrant.



Let ABC be a Quadrant, or fourth Part of a Circle, whose Radius, or Semidiameter, is 11.3, and the half Arch-line 8.875; these multiply'd together, the Product is 100.2875 for the Area.

These are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

Before I proceed to shew how to find the Area of the Sector, and Segment of a Circle, I shall shew how to find the Length of the Arch-line, both *Geometrically* and *Arithmetically*.

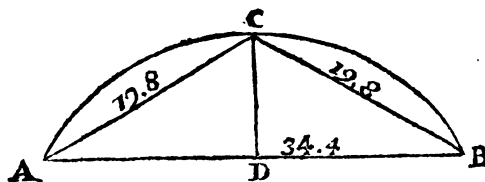
To find the Length of the Arch-line Geometrically.

Divide the Chord-line AB into four equal Parts, and set one of these Parts from B to C , and draw a Line from C to three of those Parts at D ; so shall CD be equal to half the Arch-line ACB .



To find the Length of the Arch-line Arithmetically.

Multiply the Chord of half the Segment AC or CB by 8, and from the Product subtract the Chord of the whole Segment AB , and divide the Remainder by 3; the Quotient is the Arch-line ACB sought.



$$\begin{array}{r}
 19.8 \text{ AC} \\
 \quad 8 \\
 \hline
 158.4 \\
 34.4 \text{ AB} \\
 \hline
 3 \overline{)124} \\
 \hline
 \text{Arch-line } 41.333
 \end{array}$$

Another

Another Way.

From the double Chord of half the Segment's Arch, subtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch-line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you subtract 34.4, the Remainder is 5.2, which divided by 3, the Quotient is 1.733; this added to 39.6, (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-line ACB was stretch'd out strait, it would then contain 41.333 such Parts as the Chord AB contains 34.4 of the like Parts.

These two Rules may very easily be prov'd out of the Table of natural Sines; thus,

Suppose (in the former Figure) the Arch ACB to contain 120 Degrees; the natural Sine of half, *viz.* of 60 Degrees, is 86602, which being doubled, is 173204, which is the Chord of the whole 120 Degrees, that is, AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the Half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the first Rule, multiply 100000 by 8, the Product is 800000; from which subtract 173204, (the Chord AB) and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the first Rule. Now let us examine how near this comes to the true Quantity of the Arch propos'd. If the Radius or Semidiameter of a Circle be 100000, (as it is in the Table of Sines) then the Circumference will be 628318; and because 120 Degrees is the third Part of the Circle, take the third Part of 628318 is 209439, which is the true Quantity of the Arch ACB in such Parts as

114 *Mensuration of Superficies. Part II.*

the Semidiameter contains 100000, and differs from that before found 507, which is a thing inconsiderable in *Practical Mensuration*. The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the same as above; either of the Rules gives the Quantity of the Arch-line too little, and the greater the Arch, the greater the Error. If you know the Degrees that are contain'd in the Segment's Arch, and would have the Arch-line very exactly, you may reason thus by the *Rule of Three*.

As the Circle's Periphery in Degrees, is to its Periphery in equal Parts; so is the Arch in Degrees and decimal Parts, to the same Arch in equal Parts.

Suppose the Circumference of a Circle be 71, and suppose the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then say,

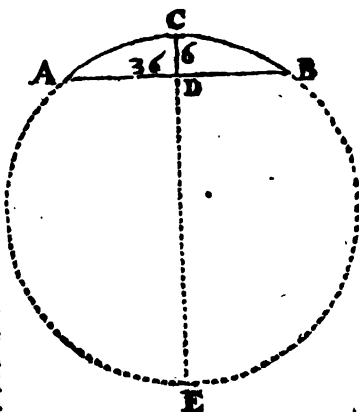
Deg. Parts	Deg.
As 360 : 71 ::	52.25
	<u>71</u>
	5225
	<u>36575</u>
	36 0)370 9.75(10.305 <i>fers</i>
	<u>36</u>
	109
	<u>108</u>
	175

So the 52 Degrees 15 Minutes will contain 10.305 of such Parts as the Circumference contains 71.

Thus have I shew'd several Ways of finding the Measure of the Curve-line of any Part of a Circle very near the Truth. The next thing I shall shew, is,
How

How to find the Diameter of a Circle by having the Chord and versed Sine of the Segment Arithmetically.

Because the Chord AB cuts the Diameter EC at right Angles, therefore the Semichord AD or DB is a mean proportional Line between the Parts of the Diameter CD and DE (by *Euc. 6. 13.*); therefore, if you square the Semichord AD or DB , and divide that Square



by the versed Sine CD , the Quotient will be the Part of the Diameter wanting; to which add the given versed Sine CD , and the Sum is the Diameter sought.

Example. Let ACB be a Segment given, whose Chord AB is 36, and the versed Sine CD 6; half 36 is 18, which squar'd, makes 324; this divided by 6, the Quotient is 54; to which add 6, the Sum is 60, the Diameter of the Circle CE .

216 *Mensuration of Superficies. Part II.*

18 half the Chord.

18

144

18

6)324 the Square of AD.

54 the Part wanting DE.

6 the versed Sine CD add.

60 the Diameter CE.



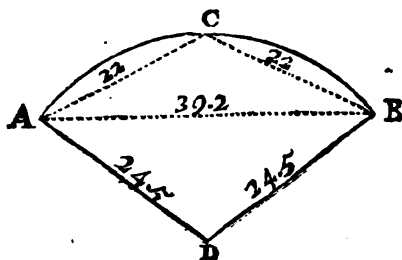
§ XII. *Of the Sector of a CIRCLE.*

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are suppos'd not to make one Right-line, and a Part of the Circumference: Whence a Sector may be either less or greater than a Semicircle. To find the Area or superficial Content thereof, this is

The RULE.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

Let ADBC be the Sector of a Circle given, whose Semidiameter AD or BD is 24.5, and the Arch-line ACB (by the first Rule, *Pag.* 112.) I find to be 45.6; the Half thereof 22.8, being multiply'd by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.

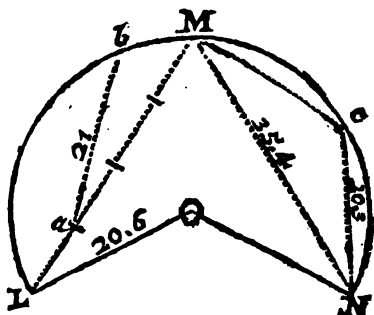


$$\begin{array}{r}
 22 \\
 8 \\
 \hline
 176 \\
 39.2 \text{ Subtrahend.} \\
 \hline
 3)136.8 \\
 \hline
 45.6 \text{ Arch-line.} \\
 \hline
 22.8 \text{ half.} \\
 24.5 \text{ Semidiameter.} \\
 \hline
 1140 \\
 912 \\
 456 \\
 \hline
 558.60 \text{ the Area.}
 \end{array}$$

Again. Let LMNO be a Sector greater than a Semicircle, whose Semidiameter LO or NO is 20.6, and the Line *ba* equal to a fourth Part of the Arch-line *LbMN* 21, the Double whereof is 42, equal to the Arch-line *LbM* or *McN*; or by the Arithmetical Rule, *Pag.* 112 the said Arch is found to be 42.333, which multiply'd by 20.6, the Semidiameter, makes 872.0598 for the Area of the Sector LMNO.

See the following Work:

20.3.



20.3 Chord N c
8

42.333
20.6

162.4
35.4 Chord MN.

253998
84666

3) 12.70

872.0598 Area.

42.333 Arch-line.



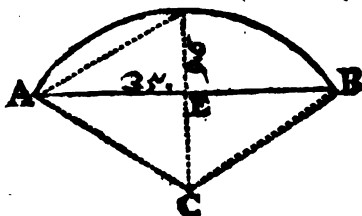
§ XIII. Of the Segment of a CIRCLE.

A Segment of a Circle is a Part terminated by a Right-line less than the Diameter, call'd a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you must, first, find the Centre of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the last Section) find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and subtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-

Otherwise you may, without describing the Figure, find the Semidiameter of the Circle by the Arithmetical Rule, pag. 113. and by the Arithmetical



Rule, pag. 112. find the Arch-line; then multiply half the Arch-line by the Semidiameter; so have you the Area of the Sector. Then subtract the versed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then subtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the Work.

$$\begin{array}{r}
 2)35=AB \\
 \hline
 17.5 \\
 17.5 \\
 \hline
 875 \\
 1225 \\
 175 \\
 \hline
 9.6)306.25(31.9 \\
 \hline
 182 \quad 9.6 \text{ add} \\
 865 \quad 41 \quad 5 \text{ the Diameter of the Circle.} \\
 \hline
 1 \quad 20.75 \text{ the Semidiameter.} \\
 9.6 \text{ DE Subtrahend.} \\
 \hline
 11.15 \text{ remains the Perpendicular EC.}
 \end{array}$$

120. *Mensuration of Superficies.* Part II.

11.15 the Perpendicular EC.

17.5 half the Chord AE or EB.

$$\begin{array}{r} 5575 \\ 7805 \\ \hline 1115 \end{array}$$

195.125 the Area of the Triangle.

306.25 the Square of AE.

92.16 the Square of DE the versed Sine.

398.41 Sum.

The square Root thereof is 19.96 the Chord AD.

$$\begin{array}{r} 8 \\ \hline 159.68 \\ \text{Sub. } 35 \text{ the Chord AB.} \end{array}$$

$$\begin{array}{r} 3 \overline{)124.6} \end{array}$$

2)41.56 the Arch-Line.

20.78 half.
20.75 Semidiameter.

$$\begin{array}{r} 10390 \\ 14546 \\ \hline 4156 \end{array}$$

From 431.1850 Area of the Sect.

Subtract 195.125 Area of the Trian

Remains 236.060 Area of the Segm.

Again,

Chap. I. Mensuration of Superficies. 121

Again, let MACBM be a Segment greater than a Semicircle; observe the former Rules, in all respects, as in the last Example; only, instead of subtracting the Area of the Triangle out of the Area of the Sector, here you must add it therunto, as may plainly appear by the following Figure.

11.5
8

92.0
20

3)72

24 Half Arch-line.

11.64 Semi-diam.
24

4656
2328

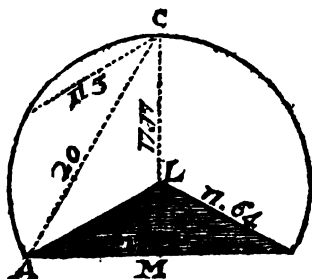
279.36 Area of the Sector LACBL.

10.25 half the Base MA.
5.53 the Perpendicular LM.

3075
5125
5125

56.6825 the Area of the Triangle ALM.
279.36 the Area of the Sector add.

336.0425 the Area of the Segment sought.



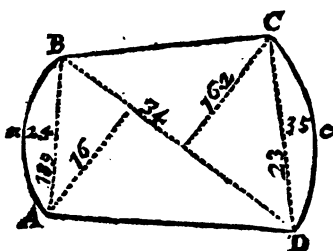
17.17
11.64

5.53 LM.

§ XIV. Of Compound Figures.

MIX'D or compound Figures are such as are compos'd of rectilineal and curvilinear Figures together.

To find the Area of such mix'd Figures, you must find the Area of the several Figures of which the whole compound Figure is composed, and add all the Area's together, and the Sum will be the Area of the whole compound Figure.



$$\begin{array}{r}
 16.2 \\
 16 \quad \left. \vphantom{16.2} \right\} \text{add.} \\
 \hline
 32.2 \text{ Sum.} \\
 \hline
 16.1 \text{ half.} \\
 34 \text{ Diagonal.} \\
 \hline
 644 \\
 483 \\
 \hline
 \end{array}$$

547.14 Area of the
(Trapezium.

10.236 half the Arch-line A a B.
14.83 Semidiameter of the Arch A B.

$$\begin{array}{r}
 30708 \\
 81888 \\
 40944 \\
 10236 \\
 \hline
 \end{array}$$

151.79988 Area of the Sector.

Chap. I. Mensuration of Superficies. 123

From 14.83 Semidiameter.

Subtract 3.4 versed Sine.

C 11.43 Perpend. of the Triangle.
 9.45 half the Chord AB:

5715
4572
10287

108.0135 the Area of the Triang. subtracted from
151.7999 the Area of the Sector.

43.7864 the Area of the Segment A a BA.

12.19 half the Arch-line C c D.
20.64 Semidiameter.

4876
7314
24380

251.6016 the Area of the Sector.

From 20.64 the Semidiameter,

Subtract 3.5 versed Sine.

Remaind. 17.14 Perpendicular of the Triangle.
 11.5 half the Chord DC.

8570
1714
1714

Sub. 197.110 Area of the Triangle.

From 251.602 Area of the Sector.

Rem. 54.492 the Area of the Segment Cc DC.

43.786 the Area of the Segment Aa BA.

547.4 The Area of the Trapezium.

Sum 645.678 the Area of the Whole.

M 2

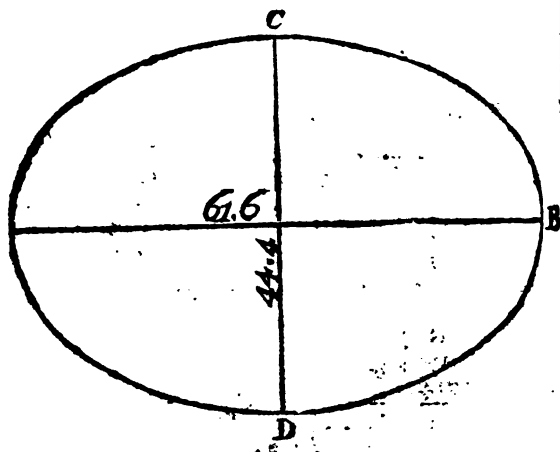
§ XV.

§ XV. Of an ELLIPSIS.

AN Ellipsis, or Oval, is a Figure bounded by a regular curve Line, returning into itself; but of its two Diameters, cutting each other in the Centre, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The RULE.

Multiply the transverse Diameter by the Conjugate, and multiply that Product by .7854, this last Product is the Area of the Oval.



Chap. 1. Mensuration of Superficies. 125

61.6 the tranſverſe Diameter.

44.4 the conjugate Diameter.

$$\begin{array}{r} 2464 \\ 2464 \\ \hline 2464 \end{array}$$

2735.04 the Rectangle.

.7854 the Area of Unity.

$$\begin{array}{r} 1094016 \\ 1367520 \\ 2188032 \\ \hline 1914528 \end{array}$$

2148.100416 the Area of the Oval.

Demonſtration. If you circumscribe any Ellipsis with a Circle, and suppose an infinite Number of Chord-lines drawn therein, all parallel to the conjugate Diameter, as those in the following Figure, then it will be,

As DA, the Diameter of the Circle, is to N n, the conjugate Diameter of the Ellipsis; so is B a B, any Chord in the Circle, to b a b, its respective Ordinate in the Ellipsis.

For, according to the Property of the Circle,

it is 1 $a S \times T a = \square B a.$

And by the Property of the Ellipsis,

it is 2 $\square T C : \square N C :: a S \times T a : \square b a.$

1, 2, 3 $\square T C : \square N C :: \square B a : \square b a.$

3, hence 4 $T C : N C :: B a : b a.$

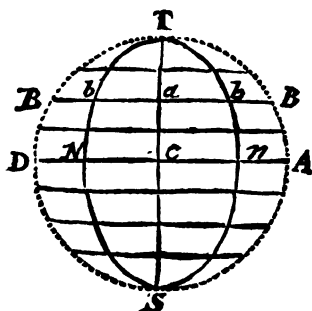
Conseq. 5 $T C : 2 N C :: 2 B a : 2 b a.$

That is 6 $DA : N n :: B a B : b a b.$

But the Sum of an infinite Series of such Chords as B a b, do constitute the Area of the Circle. And the Sum of the like Series of their respective Ordinates, as b a b, do constitute the Area of the Ellipsis:

M 3

There-



Therefore, $TS : Nn :: \text{Circle's Area} : \text{the Ellipsis Area}$. But $TS : Nn :: \square TS : TS \times Nn$; whence it follows that,

$\square TS : \text{Circle's Area} :: TS \times Nn : \text{Ellipsis Area}$.

Consequently, As 1 is to .7854; so is the Rectangle, or Product of the transverse and conjugate Diameter of any Ellipsis to its Area.

Hence it is easy to conceive, that the square Root of the Product of the transverse and conjugate Diameters will be the Diameter of a Circle equal to the Ellipsis.

Hence also all Segments of an Ellipsis, and its circumscribing Circle, (whose Bases are parallel to the conjugate Diameter, and of the same Height) are in Proportion one to another as their Bases are; that is,

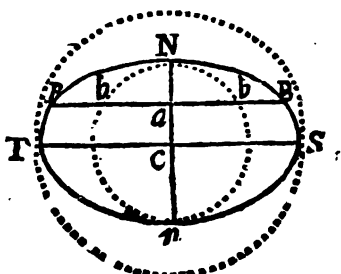
$BaB : bTb :: \text{Area Segment BTB} : \text{Area Segment bTb}$.

Or, $TS : Nn :: \text{Area Segment BTB} : \text{Area Segment bTb}$.

The Area of every Ellipsis is a mean Proportional between the Areas of its circumscribing and inscrib'd Circles.

The Truth of this may easily be reduc'd from the last, for 'tis already prov'd, that
 $\square TS : TS \times Nn ::$ circumscribing Circle's Area : Ellipsis Area.

But $\square TS : TS \times Nn :: TS \times Nn : \square Nn$. Therefore Ellipsis Area : inscrib'd Circle's Area : $TS \times Nn : \square Nn$.



Example, Let $TS=36$, and $Nn=18.4$.
 Then $\square TS=1296$, and $\square Nn=338.56$.
 Then $1296 \times .7854=1017.8784$ great Circle's Area;
 And $338.56 \times .7854=265.905$, &c. lesser Circle's Area;
 And $36 \times 18.4=662.4 \times .7854=520.24896$, which is the Area of the Ellipsis; then it will be,
 $1017.878 : 520.24896 :: 520.24896 : 265.905024$.

That is, As the great Circle's Area is to the Area of the Ellipsis; so is the Area of the Ellipsis to the Area of the lesser Circle.

From hence it follows, that all Segments of an Ellipsis, and its inscrib'd Circle, (whose Bases are parallel to the transverse Diameter, and have the same Height) are in Proportion one to another as the Area of the Ellipsis and Circle are.

That is, As the Area of the Circle is to the Area of the Ellipsis, so is the Segment $b N b$: to the Segment BNB.

Or, $Nn : TS ::$ Area Segment $b N b$: Area Segment BNB.

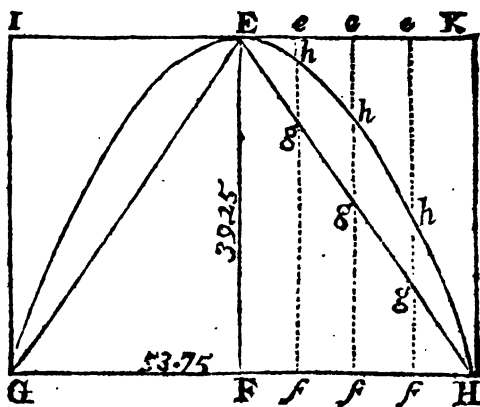
§ XVI. *Of a PARABOLA.*

A Parabola is a curvilineal Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is two Thirds of its circumscribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Base, or greatest Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the last Product by 3, the Quotient will be the Area of the Parabola.



Chap. I. Mensuration of Superficies. 129

53.75 the Ordinate GH.
39.25 the Perpendicular EF.

$$\begin{array}{r}
 26875 \\
 10750 \\
 48375 \\
 16125 \\
 \hline
 2109.9875 \\
 2 \\
 \hline
 3)4219.3750
 \end{array}$$

1406.4583 the Area.

Demonstration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, &c. and through the Divisions draw Lines, as ef, ef, &c. parallel to the Axis EF. Suppose also EF to be 4.

Then, I say, the Parabolic Space Eh HF is to the Parallelogram EK FH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, first, gf, gf, gf, &c. are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, fe : ge :: ge : he; but he in the Axis EF = 0. and in the first Parallel ef must be equal to $\frac{1}{4}$, in the next ef must be equal to $\frac{2}{4}$, in the third to $\frac{3}{4}$, and so on, in a duplicate arithmetical Progression.

For ef (=4) : ge (=1) :: ge (=1) : eh (=1)
And the second ef (=4) : eg (=2) :: eg (=2) : eh (=2), &c. And thus it will be, if the Lines Ff, ff, &c. be again bisected, &c. *ad infinitum*, so that all the Indivisibles of the trilinear Space EKHhE will be in a duplicate arithmetical Progression increasing. But the Sum of a Rank of such Terms is subtriple to a Rank of as many equal to the greatest (by Lemma 3); wherefore the whole trilinear Space EKHhE is to the Parallelogram as 1 to 3; and, consequently, the remaining parabolic Space must be to it as 2 to 3; which was to be proved.

And

130 *Mensuration of Superficies. Part II.*

And since the Triangle FEH is to the Parallelogram as 1 to 2, it must be to the Parabola as $1\frac{1}{2}$ to 2. or as 3 to 4; which was to be prov'd.

Before I proceed to the Mensuration of solid Bodies, I will lay down such Lemma's as will be necessary to facilitate the Demonstration of all such Solids.

LEMMA I.

In any Series of equal Numbers (representing Lines or other Quantities) as 1, 1, 1, 1, &c. or 2, 2, 2, 2, &c. or 3, 3, 3, 3, &c. if one of the Terms be multiply'd into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progressions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, &c. (representing a Series of Lines or Roots beginning with a Point) if the last Term be multiply'd into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the last Term, N = the Number of Terms, and S = the Sum of all the Series; then will $NL = 2S$; consequently, $\frac{1}{2}NL = S$, viz. One-half of so many times the greatest Term as there are Numbers of Terms in the Series.

$$\text{Thus } \left\{ \begin{array}{l} 0+1+2+3+4=10 \text{ the Sum} = \frac{1}{2}NL. \\ 4+4+4+4+4=20=NL. \end{array} \right.$$

LEMMA III.

If a Series of Squares, whose Sides or Roots are in arithmetical Progression, beginning with a Cypher, &c. be infinitely continued, the last Term, being multiply'd into the Number of Terms, will be triple to

Chap. 1. *Mensuration of Superficies.* 131

to the Sum of all the Series, viz. $NLL=3S$; or, $\frac{1}{3} NLL=S$.

That is, the Sum of such a Series will be One-third of the last or greatest Term, so many times repeated as there are Numbers of Terms in their Series.

Instances in square Numbers.

$$1 \quad \left\{ \begin{array}{c} 0+1+4 \\ 4+4+4 \end{array} \right. = 1\frac{1}{2} = \frac{1}{2} + 1\frac{1}{2}.$$

$$2 \quad \left\{ \begin{array}{c} 0+1+4+9 \\ 9+9+9+9 \end{array} \right. = 1\frac{2}{3} = \frac{1}{3} + 1\frac{2}{3}.$$

$$3 \quad \left\{ \begin{array}{c} 0+1+4+9+16 \\ 16+16+16+16+16 \end{array} \right. = 1\frac{3}{4} = \frac{1}{4} + 1\frac{3}{4}. \quad (\&c.)$$

From these Instances it is evident, that as the Number of Terms in the Series do increase, the Fraction or Excess above $\frac{1}{3}$ does increase, the said Excess always being $\frac{1}{6N-6}$; which, if we suppose the Series to be infinitely continued, will quite vanish, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whose Roots are in arithmetical Progression, beginning with a Cypher, &c. (as above) be infinitely continued, the Sum of all the Series will be $\frac{1}{4} NLLL=S$.

That is, One-fourth of the last Term so many times repeated as there are Numbers of Terms.

Instances

Instances in cube Numbers.

If 0, 1, 2, 3, 4, 5, &c. be the Roots of the Cubes,

$$\begin{aligned}
 1 \quad & \left\{ \frac{0+1+8+27}{27+27+27+27} = \frac{38}{108} = \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \right. \\
 2 \quad & \left\{ \frac{0+1+8+27+64}{64+64+64+64+64} = \frac{100}{320} = \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \right. \\
 3 \quad & \left\{ \frac{0+1+8+27+64+125}{125+125+125+125+125+125} = \frac{225}{750} = \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \right.
 \end{aligned}$$

From these Examples it plainly appears, that as the Number of Terms in the Series increases, the Fraction or Excess above $\frac{1}{4}$ decreases, the Excess being always $\frac{1}{4N-4}$; which, if we suppose the Series to be infinitely continued, will become infinitely small or nothing.

LEMMA V.

If a Series of Biquadrates, whose Roots are in arithmetical Proportion, beginning with a Cypher, &c. as before, be infinitely continued, the Sum of all the Terms in such a Series will be $\frac{1}{4}$ NLLLL.

The Truth of this may be manifest by the like Process as in the foregoing Lemma's, and so on for higher Powers.

LEMMA VI.

The Sum of an infinite Progression, whose greatest Term is a square Number, the others decreasing by odd Numbers, viz. 1, 3, 5, &c. is in subsequaliteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Chap. I. *Mensuration of Superficies.* 133

Instances in such Progressions.

- 1 $\left\{ \frac{9+8+5}{9+9+9} = \frac{22}{27} = \frac{2}{3} + \frac{2}{27} \right.$
- 2 $\left\{ \frac{16+15+12+7}{16+16+16+16} = \frac{50}{64} = \frac{2}{3} + \frac{1}{32} \right.$
- 3 $\left\{ \frac{25+24+21+16+9}{25+25+25+25+25} = \frac{95}{125} = \frac{2}{3} + \frac{2}{25} \right.$
- 4 $\left\{ \frac{36+35+32+27+20+11}{36+36+36+36+36+36} = \frac{161}{216} = \frac{2}{3} + \frac{1}{216} \right.$

From these Examples it plainly appears, that as the Number of Terms in the Series increases, the Fraction or Excess above $\frac{2}{3}$ decreases; and if we suppose the Series to be infinitely continued, that Excess will quite vanish, and the Sum of the infinite Series will be $\frac{2}{3}$ of so many equal to the greatest.





C H A P. II.

The Menfuration of SOLIDS.

SOLID Bodies are fuch as do confift of Length, Breadth, and Thicknefs; as Stone, Timber, Globes, Bullets, &c.



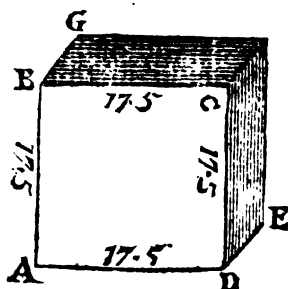
§ I. Of a CUBE.

A CUBE is a fquare Solid, comprehended under fix geometrical Squares, being in the Form of a Dye. To find the folid Content, this is

The RULE.

Multiply the Side of the Cube into itfelf, and that Product again by the Side; the laft Product will be the Solidity, or folid Content of the Cube.

$$\begin{array}{r}
 17.5 \\
 17.5 \\
 \hline
 875 \\
 1225 \\
 175 \\
 \hline
 306.25 \\
 17.5 \\
 \hline
 153125 \\
 214375 \\
 30625 \\
 \hline
 \end{array}$$

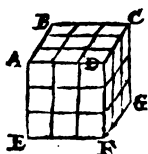


5359.375 the solid Content of the Cube.

Suppose ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiply'd by 17.5, the last Product is 5359.375, which is 5359 solid Inches and 375 Parts. To reduce the solid Inches to Feet, divide by 1728, (because so many cubical Inches is a Foot) and the solid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compasses.

Extend the Compasses from 1 to 17.5; that Extent turn'd over twice from 17.5 will reach to 5359, the solid Content in Inches. Then extend the Compasses from 1728 to 1; that Extent, turn'd the same way from 5359, will reach to 3.1 Feet.

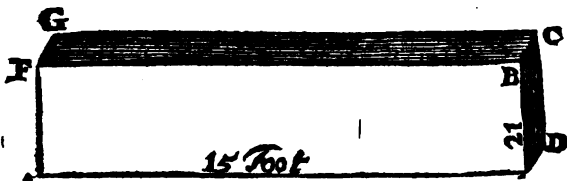


Demonstration. If the Square ABCD be conceiv'd to be mov'd down the Plain ADEF, always remaining parallel to itself, there will be generated, by such a Motion, a Solid, having six Plains, the two opposite whereof will be equal and parallel to each other; whence it is called a Parallelopipedon, or square Prism. And if the Plain ADEF be a Square equal to the generating Plain ABCD, then will the generated Solid be a Cube. From hence such Solids may be conceived to be constituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms, Therefore, if the Area of ABCD be multiply'd into the Number of Terms AE, the Product is the Sum of all the Series, (*per* Lemma I.) and, consequently, the Solidity of the Parallelopipedon or Cube. Or, if the Base ABCD, being divided into little square Areas, be multiply'd into the Height AE, divided by a like Measure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Base multiply'd by the Number of Divisions the Side AE contains. Thus, if the Side of the Base AB be 3, that multiply'd into itself is 9, which is the Area of the square Base ABCD; then, if AE be likewise 3, multiply 9 by 3, and the Product is 27; and so many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonstration it is very plain, that if you multiply the Area of the Base of any Parallelopipedon into its Length or Height, that Product will be the solid Content of such a Solid.

§ II. *Of a PARALLELOPIPEDON.*

LET ABCDEFG be a Parallelopipedon, or square Prism, representing a square Piece of Timber or Stone, each Side of its square Base ABCD being 21 Inches, and its Length AE 15 Foot.



First, then, multiply 21 by 21, the Product is 441, the Area of the Base in Inches; which multiply'd by 180, the Length in Inches, and the Product is 79380, the solid Content in Inches. Divide the last Product by 1728, and the Quotient is 45.9, that is, 45 solid Feet and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the same as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet 0 Inches 9 Parts; this multiply'd again by 15 Feet, gives 45 Feet 11 Inches 3 Parts, that is, 45 Feet and $1\frac{1}{2}$ of a Foot and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work of all these.

21	441	F. I.
21	15	1—9
—	—	1—9
21	2205	—
42	441	1—9
—	—	1—3—9
441	144)6615(45.9	—
180	—	3—0—9
—	855	15
35280	1350	—
441	—	45—0—0
—	54	7—6
1728)79380(45.9		3—9
6912		—
—		45—11—3
10260		
8640		
—		
16200		
15552		
—		
648		

By Scale and Compasses.

Extend the Compasses from 12 to 21, and that Extent will reach to near 46 Feet, being twice turn'd over from 15 Feet; so the solid Content is almost 46 Feet.

If the Base of the squar'd Solid be not an exact Square, but in form of a rectangle Parallelogram, the Way of measuring of it is much the same; for, first, you must find the Area of the Base by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before: Thus,

Chap. 2. *Mensuration of Solids.* 139

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many solid Feet are contained therein?

25	F. I.
9	2-1
<hr/>	0-9
225	<hr/>
25	1-6-9
<hr/>	25
1125	<hr/>
450	25-0-0
<hr/>	12-6-0
144)5625(39	1-0-6
432	0-6-3
<hr/>	<hr/>
1305	39-0-9
1296	
<hr/>	
09	

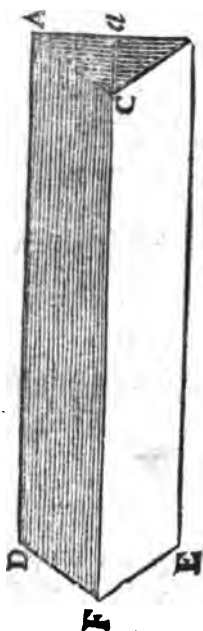
Answer 39 Feet.

By Scale and Compasses.

First, find a mean geometrical Proportion between the Breadth and the Depth; which to do upon the Line of Numbers, you must divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the mean Proportional sought: Thus the middle Point between 25 and 9 is at 15; so is 15 a mean Proportional between 9 and 25, for. $9 : 15 :: 15 : 25$; so a Piece of Timber of 15 Inches square is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compasses from 12 to 15, that Extent, turn'd twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§ III. Of a Triangular PRISM:

A Prism is a Solid contained under several Plains, and having its Bases like, equal, and parallel. The solid Content of a Prism (whether triangular or multangular) is found by multiplying the Area of the Base into the Length or Height, and the Product is the solid Content.



Let ABCDEF be a triangular Prism, each Side of the Base being 15.6 Inches, the Perpendicular thereof C a is 13.51 Inches, and the Length of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Base; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144, and the Quotient is 14.27 Feet *ferè*, the solid Content.

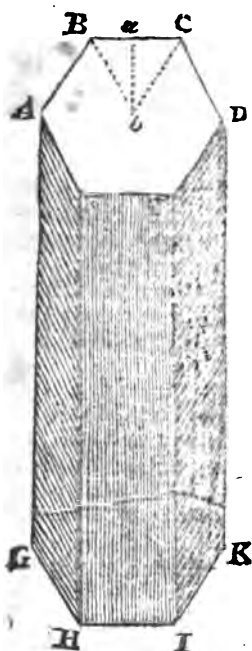
13.51	144)	2054.87	(14.27
7.8		144	
<hr/>		<hr/>	
10808		614	
9457		576	
<hr/>		<hr/>	
105.387		388	
19.5		288	
<hr/>		<hr/>	
526890		1007	
948402		1008	
105378			
<hr/>			
2054.8710			

By Scale and Compasses.

First, find a mean Proportional between the Perpendicular and Half-side, (as before taught) by dividing the Space upon the Line, between 13.51 and 7.8 into two equal Parts; so shall you find the middle Point between them to be at 10.26, which is the mean Proportional sought: By this means the triangular Solid is brought to a square one, each Side being 10.26 Inches. Then extend the Compasses from 12 to 10.26; that Extent, turn'd twice downwards from 19.5 Feet, the Length will at last fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK represent a Prism, whose Base is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular from the Centre of the Base to the Middle of one of the Sides, (ab) is 13.84 Inches, and the Length of the Prism is 15 Feet; the solid Content is required.

Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Base; (by § VIII. p. 86.) which multiply by 15 Feet, the Length, the Product is 9964.8; which divided by 144, the Quotient will be 69.2 Feet, the solid Content required.



13.84
48

11072
5536

664.32 Area of Base.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
288

...

By Scale and Compasses.

First, find a mean Proportional between the Perpendicular, and half the Sum of the Sides; that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77. Then extend the Compasses from 12 to 25.77; that Extent will reach (being twice turn'd over) from 15 Feet, the Length, to 69.2 Feet, the Content.

Chap. 2. *Mensuration of Solids.* 143

To find the superficial Content of any of the^r fore-mentioned Solids, you must take the Girth of the Piece, and multiply by the Length, and to that Product add the two Areas of the Bases, the Sum will be the whole superficial Content. Example of the hexagonal Prism last-mentioned: The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches; which multiply'd by 96, the Product is 17280 square Inches; to which add twice 664.32, the Areas of the two Bases, and the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

$$\begin{array}{r}
 180 \\
 96 \\
 \hline
 1080 \\
 1620 \\
 \hline
 17280 \\
 664.32 \\
 664.32 \\
 \hline
 144)18608.64(129.22 \\
 \hline
 420 \\
 1328 \\
 326 \\
 384 \\
 \hline
 96
 \end{array}$$

The superficial Content of the whole Solid is 129.22 Feet.

By Scale and Compasses:

Extend the Compasses from 144 to 180; that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Base, extend the Compasses from 144

144 *Mensuration of Solids.* Part II.

to 1384 ; that Extent will reach from 48 to 4.6 Feet ; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the superficial Content, as before.

The Demonstration of those last Solids will be the same as in the first Section ; for as in that, so in these, the Area of the Base is multiply'd into the Length to find the Content, and the same Reason is given for one as for the other.



§ IV. *Of a PYRAMID.*

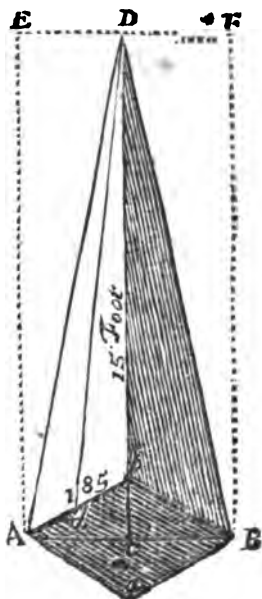
A Pyramid is a solid Figure, whose Base is a Polygon, and whose Sides are plain Triangles, their several Tops meeting together in one Point. To find the solid Content thereof, this is

The RULE.

Multiply the Area of the Base by a third Part of the Altitude, or Length ; and the Product is the solid Content of the Pyramid.

Let

Let ABD be a square Pyramid, each Side of the Base being 18.5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Base in Inches; which multiply'd by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the solid Content.



$$\begin{array}{r}
 18.5 \\
 18.5 \\
 \hline
 925 \\
 1480 \\
 185 \\
 \hline
 342.25 \text{ Area of the Base.} \\
 5 \\
 \hline
 \end{array}$$

144)1711.25(11.88 Content.

F.	I.	Pts.
1	6	6
1	6	6
<hr/>		
1	6	6
	9	3
	9	3
<hr/>		
2	4	6 3
		5
<hr/>		
11	10	7 3

By Scale and Compasses.

Extend the Compasses from 12 to 18.5 Inches, that Extent, turn'd twice over from 5 Feet, (a third Part of the Height) will fall at last upon 11.88 Feet, the solid Content.

To find the superficial Content.

Multiply the slant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Base 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the superficial Content of all but the Base; then to that add 2.38 Feet the Base, and it makes 48.69 Feet, the whole superficial Content.

180.24 the slant Height d D		
37		144)342.25(2.38
125168		288
54072		542
144)6668.88(46.31		432
576	2.38	1105
908		1152
864	48.69 the whole Content.	
448		
432		
168		
144		
24		

By

By Scale and Compasses.

Extend the Compasses from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compasses from 144 to 18.5 (one Side of the Base) that Extent will reach from 18.5 to 2.38 *fere*; which added to the other, the Sum is 48.69, the whole Superficies.

Demonstration. Every Pyramid is a third Part of the Prism, that hath the same Base and Height (by *Eucl.* 12.7.)

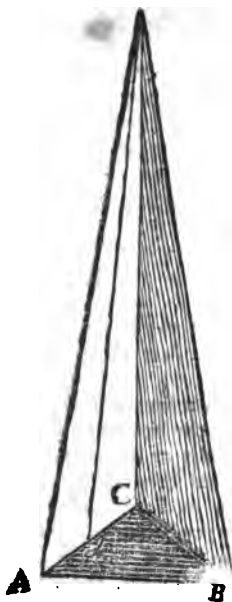
That is, the solid Content of the Pyramid ABD (in the last Figure) is one third Part of its circumscribing Prism ABEF.

For every Pyramid that hath a square Base, (such as Aa Bb in the last Figure) is constituted of an infinite Series of Squares, whose Sides or Roots are continually increasing in arithmetical Progression, beginning at the Vertex or Point D, its Base Aa Bb being the greatest Term, and its perpendicular Height CD is the Number of all the Terms: But the last Term multiply'd into the Number of Terms, the Product will be triple the Sum of all the Series (by Lemma 3.);

consequently $\frac{NLL}{3} = S$. And S is equal to the solid

Content of the Pyramid. From hence it will be easy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumscribing Prism, (that is, of a Prism of equal Base and Altitude) what Form soever its Base is of, *viz.* whether it be square, triangular, pentangular, &c. You may very easily prove a triangular Pyramid to be a third Part of a Prism of equal Base and Altitude, by cutting a triangular Prism of Cork, and then cut that Prism into three Pyramids, by cutting diagonally, as I have several times done, to satisfy myself and others.

D



Let ABCD be a triangular Pyramid, each Side of the Base being 21.5 Inches, and its Perpendicular Height 16 Feet; the Content, solid and superficial, is required.

First, find the Area of the Base, by multiplying half the Side by the Perpendicular let fall from the Angle of the Base to the opposite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, multiply'd by 21.5, the Product is 200.165 Inches, the Area of the Base. Then, because the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, which divided by 144, the Quotient is 7.41 Feet, the solid Content.

Chap. 2. *Mensuration of Solids.* 149

9.31 Half the Perpend.	F.	I.	Pts.
21.5 the Side.	Side	9	6
	Half Perp.	9	4
<u>4655</u>			
931		1	4
1862			7 2
3)200.165 Area Base.	Area Base	1	4 8 8
			4
66.72 a third Part.			
16 Height.		5	6 10 8
			4
40032			
6672		3)22	3 6 8
144)1067.52(7.41 solid Cont.	Content	7	5 2 2
1008			
595			
576			
192			
144			
48			

In casting this up by Feet and Inches, instead of multiplying by 16 the Height, I break 16 into two such Numbers, as, being multiplied together, the Product may be 16, *viz.* into 4 and 4, and multiply first by one, and then the other; a third Part of the last Product is the Content.

By Scale and Compasses.

First, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional sought. Then extend the Compasses from 12 to

O 3

14.15

14.15, that Extent (turn'd twice over from 16 Feet) will fall at last upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the superficial Content.

Multiply the slant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Base, and to that Product add the Area of the Base, the Sum is the whole superficial Content.

192.1 Inches, the slant Height d D.
Half Periph. $32.25 = 21.5 + 10.75$.

$$\begin{array}{r} 9605 \\ 3842 \\ 3842 \\ \hline 5763 \end{array}$$

6195.225 Inches, the Area of all but the Base.
200.165 Area of the Base add.

144)6395.390(44.41 Feet, the whole Content.
576

$$\begin{array}{r} 635 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 593 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 179 \\ 144 \\ \hline 35 \end{array}$$

By Scale and Compasses.

Extend the Compasses from 144. to 192.1, that Extent will reach from 32.25 (half the Periphery of the Base) to 43.02 Feet, the Content of the upper Part.

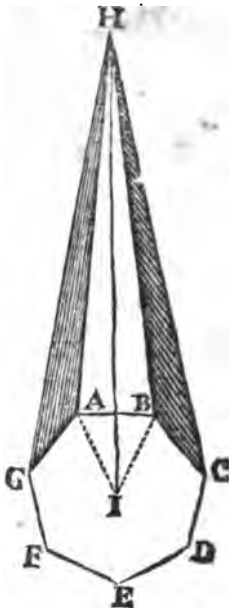
And

Chap. 2. *Mensuration of Solids.* 151

And extend the Compasses from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Base; which, added to the other, makes 44.41 Feet, the Content of the Whole.

Let ABCDEFGH be a Pyramid, whose Base is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the perpendicular Height of the Pyramid HI is 13.5 Feet; the Content solid and superficial is required.

Multiply 15.58 (the Perpendicular) by 52.5, (half the Sum of the Sides of the Heptagon) and the Product is 817.95, which multiply'd by 4.5, *viz.* $\frac{1}{2}$ of the Height, and the Product is 3680.775.



Then

152 *Mensuration of Solids.* Part II.

Then divide this last Product by 144, and the Quotient is 25.56 Feet, the Content.

15.58 the Heptagon's Perpendicular.
52.5 the Half Sum of the Sides.

$$\begin{array}{r} 7790 \\ 3116 \\ \hline 7790 \end{array}$$

817.950
4.5 a third Part of the Height.

$$\begin{array}{r} 4089750 \\ 3271800 \\ \hline \end{array}$$

144)3680.7750(25.56 solid Feet.
288

$$\begin{array}{r} 800 \\ 720 \\ \hline \end{array}$$

$$\begin{array}{r} 807 \\ 720 \\ \hline \end{array}$$

$$\begin{array}{r} 877 \\ 864 \\ \hline \end{array}$$

13

By Scale and Compasses.

First, find a geometrical mean Proportional between 15.58 and 52.5, (as is before directed) which you will find to be 28.06; then extend the Compasses from 12 to 28.06, that Extent will reach from 4.5 (twice turn'd over) to 25.56 Feet.

To find the superficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Base 162.75 Inches, by the Half-Sum of the Sides 32.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

$$\begin{array}{r}
 162.75 \\
 \underline{52.5} \\
 81375 \\
 32550 \\
 \hline
 81375
 \end{array}
 \quad
 \begin{array}{r}
 144)817.95(5.68 \\
 \underline{979} \\
 1155 \\
 \underline{3}
 \end{array}$$

144)8544.375(59.335 Feet.
5.68 Base add.
 $\begin{array}{r} 1344 \\ 483 \\ 517 \\ 855 \\ \hline 135 \end{array}$ 65.015 the whole Content.

By Scale and Compasses.

Extend the Compasses from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

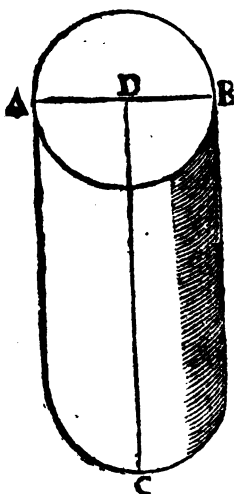
And extend the Compasses from 144 to 15.58, the Perpendicular of the Heptagon, that Extent will reach from 5.25 to 5.68 Feet, the Content of the Base; which add to the former, the Sum is 65.015, the whole superficial Content.

§ V. Of a CYLINDER.

A Cylinder is a round Solid, having its Bases circular, equal and parallel, in form of a Rollingstone used in Gardens. To find the solid Content thereof, this is

The R U L E.

Multiply the Area of the Base by the Length, and the Product is the solid Content.



Let ABC be a Cylinder, whose Diameter AB is 21.5 Inches, and the Length CD is 16 Feet; the solid Content is required.

First, square the Diameter 21.5, and it makes 462.25; which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this last Product by 144, and the Quotient is 40.34 Feet, the solid Content.

By Scale and Compasses.

Extend the Compasses from 13.54 to 21.5, the Diameter, that Extent (turn'd twice over from 16, the Length) will at last fall upon 40.34, the solid Content.

To find the superficial Content.

First, (by Chap. I. Sect. IX. Prob. 2.) find the Circumference of the Base 67.54, which multiply by 16. the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the curve Surface; to which add 5.04 Feet, the Sum of the two Bases, and the Sum is 95.09 Feet, the whole superficial Content.

$$\begin{array}{r}
 67.54 \\
 \underline{16} \\
 40524 \\
 6754 \\
 \hline
 12 \overline{) 1080.64} \quad 90.05 \} \text{ add} \\
 \underline{ 5.04} \\
 90.05 \quad 95.09
 \end{array}
 \qquad
 \begin{array}{r}
 363.05 \\
 \underline{2} \\
 144 \overline{) 726.10} (5.04 \\
 \underline{ 610} \\
 34
 \end{array}$$

By Scale and Compasses.

Extend the Compasses from 12 to 67.54, (the Circumference) that Extent will reach from 16 (the Length) to 91.05 Feet, the curve Surface.

And extend the Compasses from 12 to 21.5, (the Diameter) that Extent (turn'd twice from .7854) will at last fall upon 2.52 Feet, the Area of one Base; which doubled is 5.04; this, added to the curve Surface, makes 95.09 Feet, the whole superficial Content.

Demonstration. The solid Content of every Cylinder is found, by multiplying the Area of its Base into its Height, as aforesaid: For every right Cylinder is only a round Prism, being constituted of an infinite Series of equal Circles; that of its Base, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Base AB being multiply'd into CD,

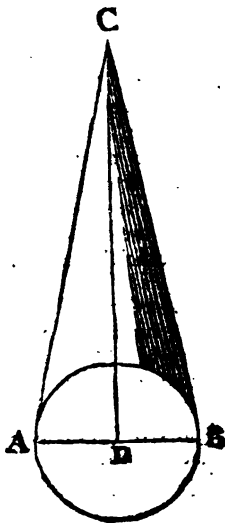
CD, will be its Solidity (by Lemma I.) Let $D=$
AB, and $H=CD$

Then .7854 DDxH=its Solidity.

A Cone is a Solid, having a circular Base, and growing smaller and smaller, till it ends in a Point which is called the Vertex, and may be nearly represented by a Sugar-loaf. To find the Solidity thereof. this is

The RULE.

Multiply the Area of the Base by a third Part of the perpendicular Height, and the Product is the solid Content.



Let ABC be a Cone, the Diameter of whose Base AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: First, square the Diameter 26.5, and it is 702.25, which multiply by .7854, and the Product is 551.54715; which multiply by 5.5, and the Product is 3033.47825; which divided by 144, the Quotient is 21-07 *fers*, the solid Content of the Cone.

26.5 the Diameter.

26.5

1325

1590

530

702.25 the Square.

.7854

280900

351125

561800

491575

551.541715 Area of the Base.

5.5

a third Part of the Height.

275773

275773

144)3033.503(21.07 Feet, the Content.

153

947

By Scale and Compasses.

Extend the Compasses from 13.54 to 26.5, the Diameter) that Extent, turn'd twice over from 5.5, (a third Part of the Height) will at last fall upon 21.07 Feet, the Content.

To find the superficial Content.

Multiply half the Circumference 41.626 by the slant Height A C 198.46, and the Product is 8261.09596; which, divided by 144, the Quotient is 57.37 *fere*, the curve Surface; to which add the Base, the Sum is 61.2, the superficial Content.

P

41.626

41.626 the Half-circumference of the Base.
 198.46 the slant Height.

$$\begin{array}{r}
 249756 \\
 166504 \\
 333008 \\
 374634 \\
 41626 \\
 \hline
 \end{array}$$

144)8261.09596(57.37 Feet *ferè.*

3.83 the Base add.

1061

530

989

61.20 the whole Content.

144)561.54(3.83

1195

434

2

By Scale and Compasses.

Extend the Compasses from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the curve Surface.

And extend the Compasses from 12 to 26.5 the Diameter; that Extent, turn'd twice over from .7854, will at last fall upon 3.83 Feet, the Base; which added to 57.37, the Sum is 61.2 Feet, the superficial Content.

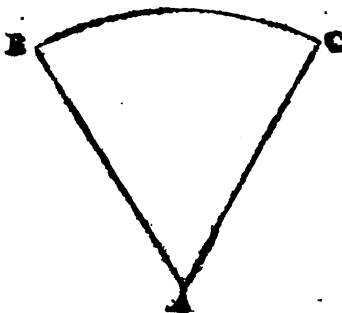
Demonstration. Every Cone is the third Part of a Cylinder of equal Base and Altitude. The Truth of this may easily be conceiv'd, by only considering, that a Cone is but a round Pyramid; and therefore it must needs have the same Ratio to its circumscribing Cylinder, as the square Pyramid hath to its circumscribing

cribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther consider'd, That

Every right Cone is constituted of an infinite Series of Circles, whose Diameters do continually increase in Arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe AB being the greateſt Term, and its perpendicular Height DC, the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiply'd by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The curve Superficies of every right Cone is equal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.

For the curve Surface of every right Cone is equal to the Sector of a Circle whose Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius AB equal to the ſlant Side of the Cone.



Which will appear very evident,

if you cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend both the Sides AB and AC together, till they meet, and you will find it to form a right Cone.

I have omitted the Demonstrations touching the Superficies of all the foregoing Solids, because I thought it needless, they being all compos'd of Squares, Parallelograms, Triangles, &c. which Figures are all demonstrated before. And if the Area of all such Fi-

- gures as compose the Solid, be found severally, and added together, the Sum will be the superficial Content of the Solid.



§ VII. *Of the Frustrum of a PYRAMID.*

A Frustrum of a Pyramid is the remaining Part, when the Top is cut off by a Plain parallel to the Base. To find the Solid Content thereof, there are several Rules.

R U L E I.

To the Rectangle (or Product) of the Sides of the two Bases add the Sum of their Squares; that Sum, being multiply'd into one third Part of the Frustrum's Height, will give its Solidity, if the Bases be square.

Or thus, which is the same in Effect :

Multiply the Area's of the two Bases together, and to the square Root thereof add the two Areas; that Sum, multiply'd by One-third of the Height, gives the Solidity of any Frustrum, square or multangled.

R U L E II.

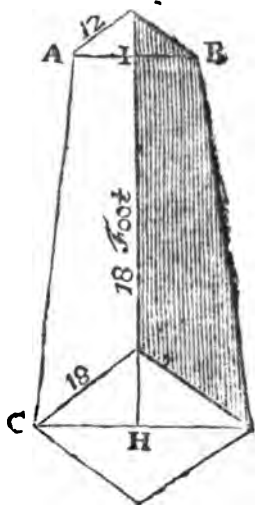
To the Rectangles of the Sides of the two Bases, add one-third Part of the Square of their Difference; that Sum being multiply'd into the Height, will produce the Solidity, if the Bases be Squares: But if they be triangular or multangular, the said Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the square Root thereof will be such a mean Side

Chap. 2. *Mensuration of Solids.* - 161

as will reduce the tapering Solid to a Prism equal thereunto.

Example. Let ABCD be the Frustum of a square Pyramid, the Side of the greater Base 18 Inches, and the Side of the lesser 12 Inches, and the Height 18 Feet; the Solidity thereof is required.

First, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whose Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Base; which multiply'd by 18 Feet the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet the Content.



Or, by the first Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of these three is 684, which multiply'd by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the same as before.

See the Work of both Ways.

$\begin{array}{r} \overbrace{18 \quad 6}^{\text{Diff.}} \\ 12 \quad 6 \end{array}$	$\begin{array}{r} \overbrace{18 \quad 12} \\ 18 \quad 12 \end{array}$
$\begin{array}{r} 216 \quad 3'35 \text{ Square} \\ 12 \text{ add} - \\ \hline 228 \text{ the Sum.} \\ 18 \text{ the Height.} \end{array}$	$\begin{array}{r} 324 \text{ Sq. } 144 \text{ Sq.} \\ 144 \\ 216 \\ \hline 624 \text{ the Sum.} \\ 6 \text{ a 3d of the Height.} \end{array}$
$\begin{array}{r} 1824 \\ 228 \\ \hline 144 \text{ } 4104 \text{ } 28.5 \\ \hline 1224 \\ 720 \\ \hline \dots \end{array}$	$\begin{array}{r} 144' + 104(28.5 \text{ Feet.} \\ \hline 1224 \\ 720 \\ \hline \dots \end{array}$

By Feet and Inches thus :

$\begin{array}{r} \text{F. I.} \\ \text{Mult. } 1 \quad 6 \\ \text{by } 1 \quad 6 \\ \hline \text{Prod. } 1 \quad 6 \quad 3'36 \text{ q.} \\ \text{add } 0 \quad 12 \\ \hline \text{Mult. } 1 \quad - \\ \text{by } 18 \quad 0 \text{ Height.} \\ \hline 18 \quad 0 \\ 9 \quad 0 \\ 1 \quad 0 \\ \hline \text{Cont. } 28 \quad 0 \end{array}$	<p style="font-size: 2em;">}</p> <p style="font-size: 1.5em;">Or thus,</p> <p style="font-size: 2em;">{</p>	$\begin{array}{r} \text{F. I.} \\ 2 \quad 3 \text{ Sq of the greater.} \\ 1 \quad 6 \text{ the Rectangle.} \\ 1 \quad 0 \text{ Sq. of the less.} \\ \hline + 9 \text{ Trip. of a mean Ar.} \\ 6 \quad 0 \text{ a 3d of the Height.} \\ \hline 28 \quad 6 \end{array}$
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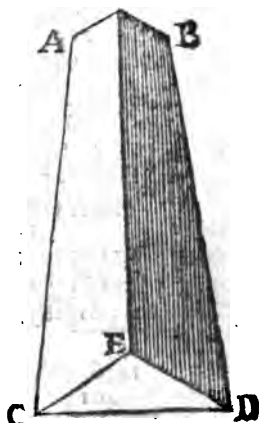
To find the superficial Content.

The Perimeter of the greater Base^A is 72, and the Perimeter of the lesser Base is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiply'd by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Bases 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole superficial Content.

18	12	18 the Height.
4	4	60
<hr/>	<hr/>	<hr/>
72	48	12) 1080
48		90 Feet.
<hr/>		2.45 the greater Base.
2) 120		1 the lesser Base.
<hr/>		<hr/>
60		93.25 the Sum.

Again; Let ABC be the Frustum of a triangular Pyramid, each Side of the greater Base 25 Inches, and each Side of the lesser Base 9 Inches, and the Length 15 Feet; the solid Content thereof is required.

By the second Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which, squar'd, makes 256, a third Part thereof is 85.333, which added to 225, the Sum is 310.333; and this multiply'd by 433, the



See the Work of both Ways.

$\begin{array}{r} \overbrace{18 \quad 6}^{\text{Diff.}} \\ 12 \quad 6 \\ \hline \end{array}$	$\begin{array}{r} \overbrace{18 \quad 12}^{\text{Diff.}} \\ 18 \quad 12 \\ \hline \end{array}$
$\begin{array}{r} 216 \quad 3)36 \text{ Square} \\ 12 \text{ add—} \\ \hline 228 \text{ the Sum.} \\ 18 \text{ the Height.} \\ \hline 1824 \\ 228 \\ \hline 144)4104(28.5 \\ \hline 1224 \\ 720 \\ \hline \dots \end{array}$	$\begin{array}{r} 324 \text{ Sq. } 144 \text{ Sq.} \\ 144 \\ \hline 216 \\ \hline 684 \text{ the Sum.} \\ 6 \text{ a 3d of the Height.} \\ \hline 144)4104(28.5 \text{ Feet.} \\ \hline 1224 \\ 720 \\ \hline \dots \end{array}$

By Feet and Inches thus :

$\begin{array}{r} \text{F. I.} \quad \text{I.} \\ \text{Mult. } 1 \quad 6 \quad 6 \\ \text{by } 1 \quad \quad 6 \\ \hline \text{Prod. } 1 \quad 6 \quad 3)36 \text{ q.} \\ \text{add } 0 \quad 1 \quad \quad \hline \quad \quad 12 \\ \hline \text{Mult. } 1 \quad 7 \\ \text{by } 18 \quad 0 \text{ Height.} \\ \hline 18 \quad 0 \\ 9 \quad 0 \\ 1 \quad 6 \\ \hline \text{Cont. } 28 \quad 6 \end{array}$	Or thus:	$\begin{array}{r} \text{F. I.} \\ 2 \quad 3 \text{ Sq of the greater.} \\ 1 \quad 6 \text{ the Rectangle.} \\ 1 \quad 0 \text{ Sq. of the less.} \\ \hline 4 \quad 9 \text{ Trip. of a mean Ar.} \\ 6 \quad 0 \text{ a 3d of the Height.} \\ \hline 28 \quad 6 \end{array}$
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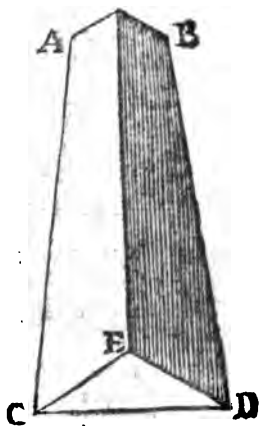
To find the superficial Content.

The Perimeter of the greater Base^d is 72, and the Perimeter of the lesser Base is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiply'd by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Bases 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole superficial Content.

18	12	18 the Height.
4	4	60
<hr/>	<hr/>	<hr/>
72	48	12) 1080
48		90 Feet.
<hr/>		2.45 the greater Base.
2) 120		1 the lesser Base.
<hr/>		<hr/>
60		93.25 the Sum.

Again; Let ABC be the Frustum of a triangular Pyramid, each Side of the greater Base 25 Inches, and each Side of the lesser Base 9 Inches, and the Length 15 Feet; the solid Content thereof is required.

By the second Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which, squar'd, makes 256, a third Part thereof is 85.333, which added to 225, the Sum is 310.333; and this multiply'd by 433, the



Product

164 *Mensuration of Solids.* Part II.

Product is 134.374, &c. which is the Area of a mean Base; and that multiply'd by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the first Rule: Find the Area of the greater Base, which you will find to be 270.625, and the Area of the lesser Base will be 35.073; these two Area's multiply'd together, the Product is 9491.630625; the Square Root thereof is 97.425; to which add the two Area's, and the Sum is 403.123; which multiply'd by a third Part of the Length 5, the Product is 2015.615; and that divided by 144, the Quotient is 13.99 Feet, as before.

See the Working of both.

$$\begin{array}{r}
 \begin{array}{r}
 25 \\
 9 \\
 \hline
 \text{Product } 225
 \end{array}
 \qquad
 \begin{array}{r}
 25 \\
 9 \\
 \hline
 16 \text{ Diff.} \\
 16 \\
 \hline
 96 \\
 16 \\
 \hline
 3)256 \text{ the Square.} \\
 85.333 \text{ a third Part.} \\
 225 \text{ add.} \\
 310.333 \\
 433 \text{ Tabular Number(vid.p.89)} \\
 \hline
 930999 \\
 930999 \\
 \hline
 1241332 \\
 \hline
 134.374189 \text{ mean Area.} \\
 15 \text{ Length.} \\
 671.870945 \\
 1343.74189 \\
 \hline
 144.2015.61 | 2835 (13.99 \text{ Feet.} \\
 575 \\
 1436 \\
 1401 \\
 \hline
 105
 \end{array}
 \end{array}$$

25	9	433
25	9	81
<hr/>	<hr/>	<hr/>
125	81 Square.	433
50		3464
<hr/>		<hr/>
625 Square.		35.073
433		
<hr/>		
1875		270.625
1875		35.073
2500		<hr/>
		811875
		1894375
270.625 Area.		13531250
		811875
		<hr/>
		9491.630625

..

9491.630625(97.425

81

187) 1391

1309

1944) 8263

7776

19482) 48706

38964

194845) 974225

974225

.....

166 *Mensuration of Solids.* Part II.

240.625 greater Area.
 97.425 the mean Proportional.
 35.073 the lesser Area.

403.123 the Triple of a mean Area.
 5 a third Part of the Height.

144)2015.615(13.99 Feet, the Solidity.
 575
 1436
 1431

 105

In finding the Area of the triangular Base, I multiply by .433, because that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Area's, or Multipliers, for finding the Area's of Polygons, you'll find is p. 89.

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the superficial Content.

The Perimeter of the greater Base is 75, and the Perimeter of the lesser Base is 27; the Sum of both is 102, and the Half-sum is 51; which multiply'd by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Bases 2.12 Feet, and the Sum is 65.87 Feet, the whole superficial Content.

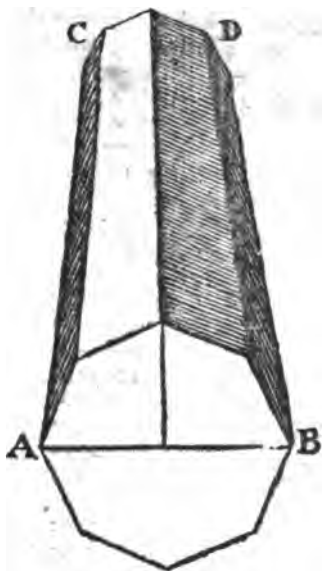
Note, That 51 should have been multiply'd by the slant Height, but the Difference it would make, is but .06 of a Foot, which is inconsiderable.

Again :

Again : Suppose ABCD to be the Frustum of a Pyramid, having an octagonal Base, each Side thereof being 9 Inches, and each Side of the lesser Base 5 Inches, and the Length or Height 10.5 Feet, the Solidity is requir'd.

By the second Rule, multiply the greater Side 9 by the lesser Side 5, and the Product is 45 ; then the Difference between 9 and 5 is 4, which squar'd makes 16 ; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333 ; multiply

this last by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Base ; which multiply'd by the Height 10.5 Feet, the Product is 2551.8066 ; then divide this last Product by 144, and the Quotient is 17.72 Feet, the solid Content.



See the Work.

Mult.

Mult. 9 Inches. 9 from the greater Side.
 by 5 Inches. 5 subtract the lesser.

Prod. 45

4

4

3 16 squar'd.

5.3333 a third Part.

Add 45

Sum 50.3333 the Square of a mean Side.

4.8284 Tabular Number, p. 89.

2013332

402666

10067

4026

201

243.0292 a mean Area.

10.5 the Height.

12151460

2430292

144)2551.80(660(17.72

144

1111

1008

1038

1008

300

288

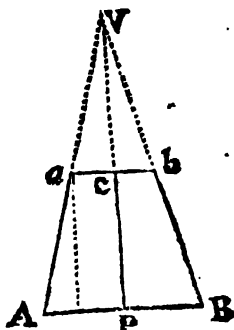
12

To find the superficial Content.

The Perimeter of the greater Base is 72, and the Perimeter of the lesser Base is 40, and their Sum is 112; the Half thereof is 56, which multiply'd by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Bases, 3.55, and the Sum is 52.55 Feet, the whole superficial Content.

Demonstration. From the Rules delivered in the IVth and VIth Sections, the two foregoing Rules may easily be demonstrated.

Suppose a square Pyramid, ABV , to be cut by a Plain at ab , parallel to its Base AB , and it were requir'd to find the Solidity of the Frustum, or Part $abAB$. Let there be given



$D=BA$, the Side of the greater Base.

$d=ba$, the Side of the lesser Base.

$H=CP$, the perpendicular Height.

First, $1 \quad D-d : H :: d : \frac{dH}{D-d} = VC \text{ by the Figure.}$

Then $2 \quad DD \times \frac{H+VC}{3} = \text{the whole Pyramid } BVA.$
by Section the IVth.

And $3 \quad dd \times \frac{1}{3} VC = \text{the Pyramid } aVb \text{ cut off.}$

Q

The

Then in the 2d and 3d Steps, if, instead of VC, you take $\frac{dH}{D-d}$ equal to it by the first Step, it will be,

$$\begin{array}{rcl}
 \text{viz. 1. 2.} & 4 & \left| \begin{array}{l} \text{DDDH} \\ \hline 3D-3d = \text{the whole Pyramid BVA.} \\ \text{dddH} \end{array} \right. \\
 \text{and 1. 3.} & 5 & \left| \begin{array}{l} \hline 3D-3d \\ \hline \text{the Pyramid a V b.} \end{array} \right. \\
 4.-5. & 6 & \left| \begin{array}{l} \text{DDDH}-\text{dddH} \\ \hline 3D-3d = \text{the Frustum abAB} \end{array} \right.
 \end{array}$$

And by dividing $DDD-ddd$ by $D-d$, and then multiplying the Quotient by $\frac{1}{3}H$, the last Step will be reduced to $DD+Dd+dd \times \frac{1}{3}H = \text{the Frustum a b A B}$, which in Words is thus;

To the Rectangle of the Sides of the two Bases add the Sum of their Squares; that Sum being multiply'd into one-third of the Frustum's Height, will give its Solidity, which is the same as the first Rule of this Section.

See the Work of the Division.

$$\begin{array}{r}
 D-d) \text{ DDD}-\text{ddd} \quad (DD+Dd+dd \\
 \text{DDD}-\text{DDd} \\
 \hline
 \text{DDd}-\text{ddd} \\
 \text{DDd}-\text{Ddd} \\
 \hline
 \text{Ddd}-\text{ddd} \\
 \text{Ddd}-\text{ddd} \\
 \hline
 \text{O} \quad \text{O}
 \end{array}$$

The same Reason will hold good for all Frustrums of Pyramids or Cones, whether the Base be triangular or multangular, because the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles are proportional to the Area, which proves the latter Part of the said first Rule.

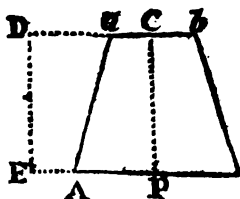
Again, to prove the second Rule.

Suppose	1	$x = D - d.$ And $F =$ the Frustrum.
then	2	$DD - \frac{1}{3} Dd + \frac{1}{3} dd = \frac{3}{H} F$ by the last.
1 \odot 2	3	$xx = DD - \frac{2}{3} Dd + \frac{1}{3} dd.$
2 $-$ 3	4	$3 Dd = \frac{3}{H} F - xx.$
4 $-$ 3	5	$Dd = \frac{F}{H} = \frac{1}{3} xx.$ Or $Dd + \frac{1}{3} xx = \frac{F}{H}.$
5 \times H	6	$(Dd + \frac{1}{3} xx) \times H = F,$ the Frustrum $abAb$

Which in Words is thus :

To the Rectangle of the Sides of the two Bases add one third Part of the Square of the Difference of the said Sides, and multiply the Sum by the Height of the Frustrum, the Product is the Solidity of the Frustrum.

The superficial Contents of Frustrums, (all but the Bases) are compos'd of Trapeziums, so many as the Frustrum has Sides. As the square Frustrum $abAb$, in the last Figure, is compos'd of four Trapeziums, having the two upper, and also the two longer Angles equal ; if therefore the Trapezium $abAB$ be cut in two by the Line CP , and the two Pieces laid together, the Line bB upon the Line aA , the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure annex'd ;



nex'd ; the Parallelogram DCEP being equal to the Trapezium $abAB$; because the Side Da is equal to PB , and EA is equal to aC . Therefore, to find the Area of the Trapezium, add half the Side ab to half the Side AB , and it makes DC or EP ; which multiply by the

Height PC , the Product is the Area of the Parallelogram $DCEP$, equal to the Trapezium $abAB$; then if that be multiply'd by the Number of Trapeziums, the Product will be the superficial Content of the Frustum, wanting the Bases. Or, if the whole Perimeter of the greater Base be added to the Perimeter of the lesser Base, and half the Sum multiply'd by Height, the Product will be the superficial Content of all the Trapeziums at once.

Note, That half the Sum of the Perimeters should be multiply'd by the slant Height, up the Middle of one of the Trapeziums ; but in the foregoing Examples I have multiply'd by the perpendicular Height, because the Difference is very inconsiderable.



§ VIII. Of the Frustum of a CONE.

A Frustum of a Cone, is that Part which remains, when the Top end is cut off by a Plain parallel of the Base. To find the solid Content, the Rules are the same in Effect as for the Frustum of a Pyramid.

RULE I.

To the Rectangle of the Diameters of the two Bases add the Squares of the said Diameters, and multiply
the

the Sum by .7854, the Product will be the Triple of a mean Area; which multiply'd by $\frac{1}{3}$ of the perpendicular Height, that Product will be the solid Content.

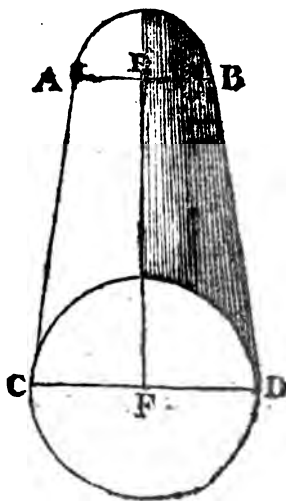
Or thus: Multiply the Area's of the greater and lesser Bases together, and out of the Product extract the Square Root, and add the two Area's and Square Root together, and multiply the Sum by One-third of the Perpendicular Height, the Product is the solid Content.

R U L E II.

To the Rectangle of the greater and lesser Diameters, add one-third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiply'd by the perpendicular Height, the Product is the Solidity.

Example. Let ABCD be the Frustum of a Cone, whose greater Diameter CD is 18 Inches, and the lesser Diameter AB 9 Inches, and the Length 14.25 Feet; the solid Content is requir'd.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whose Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiply'd by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03



Q. 3

Feet,

174 *Menfuration of Solids.* Part II.

Feet, the Area of a mean Base; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the solid Content.

Or thus, by the first Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the lesser Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of these three is 567, which multiply'd by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Base; this, multiply'd by 4.75 Feet, (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the same as before.

See the Work.

	18	18 from	.7154
	9	9 subtr.	189
	<hr/>		<hr/>
	162	9 m.	70686
Add	27	9	62832
	<hr/>		<hr/>
Sum	189	3)81 Square.	7854
		<hr/>	<hr/>
		27 a Third	144
			<hr/>
Height	14.25 Feet.		444
Area Base	1.03 Feet.		432
	<hr/>		<hr/>
	4275		12
	14250		
	<hr/>		
Solid Content	14.6775 Feet.		

324 the Square of 18.
162 the Rectangle.
81 the Square of 9.

567 the triple Square of a mean Diameter.

$$\begin{array}{r}
 .7854 \\
 \hline
 567 \\
 \hline
 54978 \\
 47124 \\
 \hline
 39270. \\
 \hline
 144)445.32(118 \quad (3.09 \\
 \hline
 1332 \quad \hline 475 \\
 \hline
 36 \quad \hline 1545 \\
 \hline
 \quad \hline 2163 \\
 \hline
 \quad \hline 1236 \\
 \hline
 \end{array}$$

The Solidity 14.6775

To find the superficial Content.

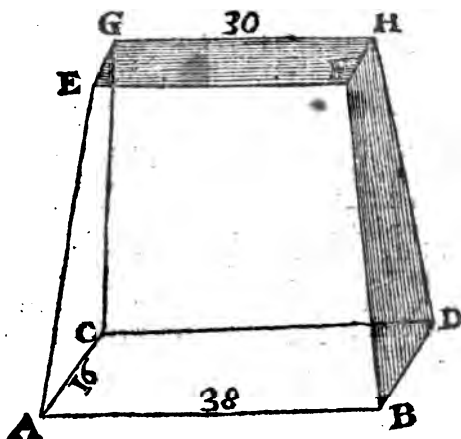
By Chap. I. Sect. IX. Problem 2. you will find the Circumference of the greater Base to be 56.5488, and of the lesser Base 28.2744; the Sum of both is 84.8232; the Half-sum is 42.4116; which multiply'd by 14.25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the curve Surface; to which add the Sum of the two Bases, 2.21 Feet, the Sum is 52.57 Feet, the whole superficial Content.

§ IX. *To measure the Frustum of a rectangled Pyramid, called a PRISMOID, whose Bases are parallel one to another, but disproportional.*

The RULE.

TO the greatest Length add half the lesser Length, and multiply the Sum by the Breadth of the greater Base, and reserve the Product.

Then, to the lesser Length, add half the greater Length, and multiply the Sum by the Breadth of the lesser Base; and add this Product to the other Product reserv'd, and multiply that Sum by a third Part of the Height, and the Product is the solid Content.



Example

Chap. 2. *Mensuration of Solids.* 177

Example. Let ABCDEFGH be a Prismoid given, the Length of the greater Base AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the lesser Base EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the solid Content is required.

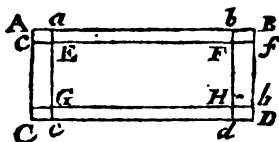
To the greater Length AB 38, add half EF the lesser Length 15, the Sum is 53; which multiply'd by 16, the greater Breadth, and the Product is 848; which reserve.

Again, to EF 30, add half AB 19, and the Sum is 49; which multiply by 12, (the lesser Breadth EG) the Product is 588; to which add 848, (the reserv'd Product) and the Sum is 1436; which multiply'd by 2, (a third Part of the Height) and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the solid Content.

$ \begin{array}{r} 38 = AB \\ 15 = \frac{1}{2} EF \\ \hline 53 \\ 16 = AC \\ \hline 318 \quad \bullet \\ 53 \\ \hline 848 \\ 588 \\ \hline 1436 \\ 2 = \text{a third Part of the Height.} \\ \hline 144 \overline{) 2872} \text{ (19.94 Feet, the Content.)} \\ \hline 1432 \\ 1360 \\ 640 \\ \hline 64 \end{array} $	$ \begin{array}{r} 30 = EF \\ 19 = \frac{1}{2} AB \\ \hline 49 \\ 12 = EG \\ \hline 588 \end{array} $
--	---

178 *Mensuration of Solids.* Part II.

To prove this Rule, Let us suppose the Solid cut into Pieces, so as to make it capable of being measured by the foregoing Rules, thus; let ABCD represent the greater Base, and EFGH the lesser Base; and let the Solid be suppos'd to be cut thro' by the Lines, *ac*, *bd*, and *ef*, *gb*, from the Top to the Bottom; so will



there be a Paralleloipedon, having its Bases equal to the lesser Base EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length

of the Base by 12, the Breadth thereof) and the Product is 360; which multiply'd by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whose Bases are *abEF*, and *GHcd*; if these two Pieces be laid together, the thick End of one to the thin End of the other, they will compose a rectangled Parallelopipedon; which to measure, multiply the Length of the Base 30 by its Breadth 2, and the Product is 60; which multiply'd by 6, (the Height) the Product is 360. Then there are two other Wedge like Pieces, whose Bases are *eEgG*, and *FfHh*; these two laid together, will compose a rectangled Parallelopipedon; to measure this, multiply the Length of the Base 12 by the Breadth 4, the Product is 48; which multiply'd by 6, (the Height) the Product is 288. And lastly, there are four rectangled Pyramids, at each Corner one; which to measure, multiply the Length of one of the Bases 4 by its Breadth 2, the Product is 8; which multiply'd by 2, (a third Part of the Height) the Product is 16; and that multiply'd by 4, (because there are four of them) the Product is 64. Then add all these together, and the Sum is 2872; and divide by 144, the Quotient is 19.94 Feet, the same as before, which shews the Rule to be true.

See

See the Work.

12	30	12	4
30	2	4	2
<hr/>	<hr/>	<hr/>	<hr/>
360	60	48	8
6	6	6	2
<hr/>	<hr/>	<hr/>	<hr/>
2160	360	288	16
360			4
288			<hr/>
64			64
<hr/>			

144)2872(19.94 Feet, the whole Content.

1432
1360
640

64

To find the superficial Content.

Half the Perimeter of the greater Base is 54, and half the Perimeter of the lesser Base is 42 ; which added together, the Sum is 96 ; which multiply'd by 6, (the Height) the Product is 576 : Divide this Product by 12, the Quotient is 48 Feet ; to which add the Sum of the two Bases 6.72 Feet, and the Sum is 54.72 Feet, the whole superficial Content.

§ X. To measure a CYLINDROID; that is, a Frustum of a Cone, having its Bases parallel to each other, but unlike.

THE RULE.

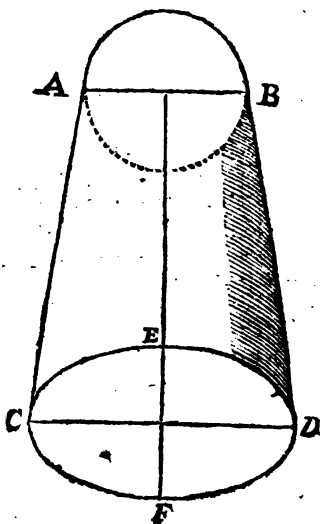
TO the longest Diameter of the greater Base, add half the longest Diameter of the lesser Base, and multiply the Sum by the shortest Diameter of the greater Base, and reserve the Product.

Then, to the longest Diameter of the lesser Base, add half the longest Diameter of the greater Base, and multiply the Sum by the shortest Diameter of the lesser Base, and add the Product to the former reserv'd Sum, and that Sum will be the triple Square of a mean Diameter; which multiply'd by .7854, and

that Product multiply'd by a third Part of the Height, the Product is the solid Content.

Examp. Let ABCD be a Cyllindroid, whose Bottom-base is an Oval, the transverse Diameter being 44 Inches: and the conjugate Diameter 14 Inches: and the upper Base is a Circle, whose Diameter is 26 Inches, and the Height of the Frustum is 9 Feet; the Solidity is requir'd,

To 44 (the greater Diameter of the lower Base) add 13, half the



Chap. 2. Mensuration of Solids.

181.

the Diameter of the greater Base) the Sum is 57; which multiply'd by 14, (the conjugate Diameter of the greater Base) the Product is 798; which reserve. Then to 26 (the Diameter of the lesser Base) add 22, (half the transverse Diameter of the greater Base) and the Sum is 48; which multiply'd by 26, (the Diameter of the lesser Base) the Product is 1248; to which add the former reserv'd Product, the Sum is 2046; which multiply'd by .7854, the Product is 1606.9284; which multiply'd by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the solid Content.

See the Work.

$$\begin{array}{r} 44 = CD \\ 13 = \text{half } AB \\ \hline 57 \text{ Sum} \\ 14 = EF \end{array}$$

228

57

798 Product reserv'd.

$$\begin{array}{r} 26 = AB \\ 22 = \text{half } CD \\ \hline 48 \text{ Sum.} \\ 26 = AB. \end{array}$$

288

96

1248

798 add.

2046

.7854

8184

10230

16368

14322

1606.9284

3

344)4820.78|52(33.47

500

687

1118

110

R

This

* This Rule being the same as that in the last Section, the Proof of that may serve as a sufficient Proof of this, if what has been before written be well considered.

To find the superficial Content.

To the Periphery of the Ellipsis 97.41, add the Periphery of the Circle 81.68, and the Sum is 179.09; the half thereof 89.545, multiply'd by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the curve Surface: Then the Area of the Ellipsis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the curve Surface, the Sum is 74.2 Feet, the whole superficial Content.



§ XI. Of a SPHERE or GLOBE.

A Sphere, or Globe, is a round solid Body, every Part of whose Surface is equally distant from a Point within it, called its Centre; and it may be conceived to be form'd by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

The RULE,

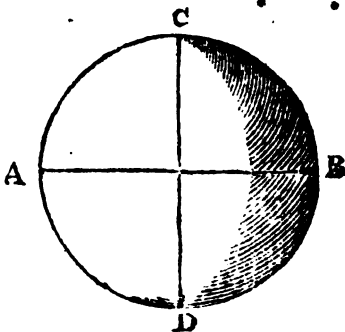
1. Multiply the Axis, or Diameter, into the Circumference, the Product is the superficial Content; which multiply by a sixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 is to 11; so is the Cube of the Axis to the solid Content.

3. Or, As 1 is to .5236; so is the Cube of the Axis to the solid Content.

Example.

Example. Let ABCD be a Globe whose Axis is 20 Inches, then the Circumf. will be 62.832 : Then, by the first Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the superficial content in Inches ;



take a sixth Part thereof, which is 209.44, (because an exact sixth Part of 20 cannot be taken) multiply that sixth Part by 20 (the Axis), and the Product is 4188.8, the Solidity in Inches. Qr, if you multiply the superficial Content by the Axis, and take a sixth Part of the Product, the Answer will be the same.

Or thus, by the second Rule :

The Cube of the Axis is 8000, which multiply'd by 11, the Product is 88000 ; which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule :

If the Cube of the Axis be multiply'd by .5236, the Product is 4188.8, the Solidity, the same as by the first Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

$$\begin{array}{r} 62.832 \\ 20 \\ \hline \end{array}$$

6)1256.640 the superficial Content.

$$\begin{array}{r} 209.44 \text{ a sixth Part.} \\ 20 \\ \hline \end{array}$$

4188.80 the Solidity in Inches.

$$\begin{array}{r} 21 : 11 :: 8000 \\ 11 \\ \hline \end{array}$$

21.88000(4190.47 the Content.

$$\begin{array}{r} 40 \\ 190 \\ 100 \\ 160 \\ \hline 43 \end{array}$$

$$\begin{array}{r} 1 : .5236 :: 8000 \\ 8000 \\ \hline \end{array}$$

1728)4188.8000(2.424 Feet, the Solidity.

$$\begin{array}{r} 7328 \\ 4160 \\ 7040 \\ \hline 128 \end{array}$$

Note. If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

By Scale and Compasses.

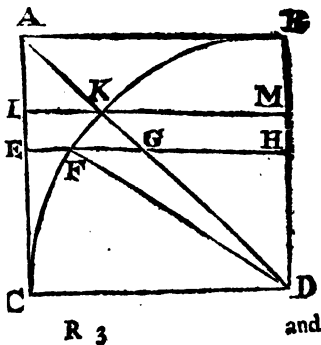
Extend the Compasses from 1 to 20 (the Axis), that Extent (turn'd three times over, from .5236), will at the last fall upon 4188.8, the solid Content in Inches. Or, Extend the Compasses from 1728 to 8000, (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the solid Content in Feet.

Extend the Compasses from 1 to 20 (the Axis), that Extent (turn'd twice over from 3.1416) will at last fall upon 1256.64, the superficial Content in Inches : Or, Extend the Compasses from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the superficial Content in Feet.

Demonstration. Every Sphere is equal to a Cone, whose perpendicular Axis is the Radius of the Sphere, and its Base a Plain, equal to all the Surface of it.

For you may conceive the Sphere to consist of an infinite Number of Cones, whose Bases, taken all together, compose the Surface, and whose Vertexes meet all together in the Centre of the Sphere : Hence the Solidity of the Sphere will be gain'd, by multiplying its Surface by $\frac{1}{3}$ of its Radius.

Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be supposed all three to revolve round the Line BD as an Axis: Then will the Square generate a Cylinder, the Quadrant a Hemisphere,



and the Triangle a Cone, all of the same Base and Altitude.

Then the Square of $EH (= \square FD) = \square FH + \square DH$ (but $DH = GH$). And since Circles are as the Squares of their Diameters, (by *Euc.* 12. 2.) the Circle made by the Revolution of EH must be equal to both the Circles made by the Motions of FH and GH .

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH , equal to the Ring describ'd by the Motion of EF . And thus it will always be, where-ever you draw the Line EH or IM , &c.

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of the EF 's, must be equal to that of all the Circles made by the Motion of the GH 's, *i. e.* the Dish-like Solid, form'd by the revolving Rings, will be equal to the Cone, form'd by the Revolution of the GH 's, which are the Elements of the Triangle ABD ; that is, the Dish-like Solid will be as the Cone, $\frac{1}{3}$ of the circumscribing Cylinder, and consequently the Hemisphere must be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder.

Let the Radius of the Sphere be $r = CD$, then the Diameter will be $2r$; let the Surface of the Sphere, generated by the revolving Semicircle, be called S , and that of the Cylinder, form'd by the Revolution of $2AC = 2r = \text{Diameter}$, be called f . Wherefore in what was just now prov'd, the Expression for the Solidity of the Sphere in this Notation, will be $\frac{1}{3} r S$; and putting c equal to the Circumference of the Base, or for the Periphery of a great Circle of the Sphere, the curve Surface of the Cylinder will be $2rc$, also $\frac{rc}{2}$ will be the Area of a great Circle, (by Sect. IX. of Chap. I. Prob. 1.) and this multiply'd by $2r$, makes $2rc$, which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, since f was put equal to $2rc =$
the

the curve Surface of the Cylinder, $\frac{f r}{2}$ (by substituting f for $2 r c$) will be also $=$ the Solidity of the Cylinder. Now, since the Sphere is $= \frac{2}{3}$ of the Cylinder, $\frac{r S}{3} = \frac{2 \times f r}{3 \times 2}$; that is, $\frac{r S}{3} = \frac{2 f r}{6} = \frac{f r}{3}$. Wherefore $r S = r f$, that is, dividing by r , $S = f$; or the Surface of the Sphere is equal to the curve Surface of the Cylinder, but the curve Surface of the Cylinder was $2 r c$.

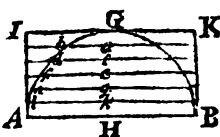
Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you must multiply the Diameter ($= 2 r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Base. From this Notation also $\frac{r c}{2}$, the Area of a great Circle of

the Sphere is plainly $\frac{1}{4}$ of $2 r c$, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greatest Circle of it.

Wherefore, to $2 r c$, the convex Surface of the Cylinder, add $r c$, equal to the Area of both its Bases, you will have $3 r c$; which shews you, that the Surface of the Cylinder (including its Cases) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the same Base and Altitude, by Lemma VI. foregoing, thus:

Let A G B represent the Hemisphere, and A I K B half the Cylinder; then, if the Semi-diameter G H be divided into six equal Parts, and Lines drawn parallel to A B, the Diameter,



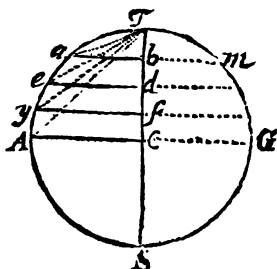
the Squares of the Semichords, ab , cd , ef , &c. will be

be a Series of Numbers, whose greatest Term AH is a square Number, the other differing by odd Numbers; that is, AH is 36, kl 35, gh 32, ef 27, cd 20, ab 11: But an infinite Series of such Numbers are in Proportion to an infinite Number of Terms equal to the greatest, as 2 to 3. And because the Hemisphere is compos'd of an infinite Number of Circles, whose Diameters are the Chords of the Semicircle; and the Half-cylinder is compos'd of an infinite Number of Circles, whose Diameters are all equal to the Diameter of the Semicircles AB; therefore the Hemisphere is in Proportion to the Half-cylinder, as 2 to 3; and consequently, the whole Sphere bears the same Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greatest Circle, is thus proved:

The Solidity of the Sphere is constituted of an infinite Number of parallel Circles (as is aforesaid); consequently, the Superficies of the Sphere will be compos'd of the Peripheries of those Circles which constitute its Solidity.

Note, In the following Demonstrations, © signifies any Circle in general; and if any two Letters be joined to it, thus, © AB, &c. then it denotes the Area of such a Circle as those two Letters represent the Radius of.



Let $D=TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be	1	$D - Tb \times Tb = \square ab;$
that is,	2	$D \times Tb = \square Tb = \square ab;$
therefore	3	$D \times Tb = \square aT.$

For	4	$\square ab + \square Tb = \square aT$ (<i>Euc. 1. 47.</i>)
and {	5	$D \times Td = \square eT.$
	6	$D \times Tf = \square yT.$

Hence it is evident, that the Series $\square aT, \square eT, \square yT, \&c.$ are in the same Ratio with $Tb, Td, Tf, \&c.$ viz. in arithmetical Progression: Whence it follows, that the $\odot aT$ = to the Sum of all the Circle's Peripheries, between T and b .

And $\odot eT$ = the Sum of all the Circle's Peripheries between T and d , $\&c.$

Consequently, that the $\odot AT$ = the Sum of all the Circle's Peripheries, included between T and C ; that is, $\odot AT$ = the Superficies of the Hemisphere.

And because $\square AC + \square TC = \square AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT = 2 \odot AC$, is the Superficies of the Hemisphere.

Consequently, $4 \odot AC$ will be the Superficies of the whole Sphere. Which was to be proved.

Scholium.

From the Method here used in proving the whole Superficies, it will be easy to find the curve Superficies of any Fruustum, or Part of a Sphere, that is cut off by a right Line, or Plain, *viz.* such as the Fruustum a T m in the last Scheme, whose curve Superficies is $\odot a T$, as above. Therefore (because $\square a b + \square T b = \square a T$) it will be $\odot a b + \odot T b =$ the curve Superficies of that Fruustum.

But if the Axis TS, and the Height T b of the Fruustum, are given, then it will be $T S \times T b = \square a T$, as in the third Step above, which gives the Proportion or Theorem following, *viz.*

As the Axis of the Sphere is to the whole Superficies of the Sphere; so is the Height of any Fruustum to its curve Superficies.

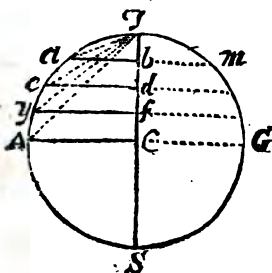
To which if there be added the Area of the Fruustum's Base, the Sum will be the whole Superficies of the Fruustum.

That the Solidity of every Sphere is Two-thirds of its circumscribing Cylinder, may be thus proved.

According to the Work above, it appears, that $\odot a b$, $\odot e d$, $\odot y f$, &c. do constitute the Solidity of the Sphere; and that $\square a T$, $\square e T$, $\square y T$, &c. are

a Series of Terms in Arithmetical Progression, $\square A T$ being the greatest Term, and T C the Number of Terms; therefore $\odot A T \times \frac{1}{2} T C =$ the Sum of all the Series, by Lemma 2.

And because $\square a T = \square T b = \square a b$. $\square e T = \square T d = \square e d$. $\square y T = \square T f = \square y f$. $\square A T = \square T C = \square A C$, &c.
wherein



wherein $\square T b$, $\square T d$, $\square T f$, &c. are a Series of Squares, whose Roots $T b$, $T d$, $T f$, are in arithmetical Progression; $\square TC$ being the greatest Term, and TC the Number of Terms; therefore $\odot TC \times \frac{1}{2} TC =$ the Sum of all the Series, by Lemma 3.

Consequently, $\odot AT \times \frac{1}{2} TC = \odot TC \times \frac{1}{2} TC =$ the Sum of all the Series $\odot ab$, $\odot ed$, $\odot yf$, &c. which constitute the Solidity of the Half-sphere ATG . Put $D = 2 TC$ the Axis of the Sphere; then $\frac{1}{2} D = \frac{1}{2} TC$, and $\frac{1}{2} P = \frac{1}{2} TC$. And because $\square AT = 2 \square TC$, therefore $\odot AT = 2 \odot TC = 1.5708 DD$; and $1.5708 DD \times \frac{1}{2} D = 0.3927 DDD$.

Again, $\odot TC \times \frac{1}{2} TC = 0.7854 DD \times \frac{1}{2} D = .1309 DDD$, then $0.3927 DDD - 0.1309 DDD = 0.2618 DDD$, the Solidity of the Half-sphere.

Consequently, $0.2618 DDD \times 2 = .5236 DDD$ will be the solid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whose Base, and its Height, is $= D$.

For $0.7854 DDD =$ the Solidity of the Cylinder by Sect. V. But $\frac{2}{3}$ of $0.7854 DDD = 0.5236 DDD$, as before.

Scholium.

From this Demonstration it will be easy to deduce or raise Theorems for finding the solid Content of any Frustum of a Sphere, as $T m$ in the last Figure.

For we there suppose the Frustum a $T m$ to be constituted of an infinite Series of Circles, which have the same Ratio with all those Circles that constitute the Half-sphere.

Therefore it follows, that $\odot a T \times \frac{1}{2} T b : - \odot b T \times \frac{1}{2} T b$ will be the Sum of all the Circles intercepted between T and b ; consequently it will be the Solidity of that Frustum.

And because $\square a b + \square T b = a T$; therefore $\odot ab + \odot T b \times \frac{1}{2} T b : - \odot T b \times \frac{1}{2} T b =$ the Solidity. Let $c = a b$ half the Diameter of the Frustum's Base $h = T b$ its Height; and $S =$ the Solidity of the Frustum. Then $\odot ab = 3.141600$, and $\odot T b = 3.1416 h h$; consequently,

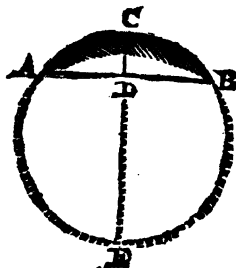
sequently, $3.1416cch + 3.1416hhh - 3.1416hhh = S.$

Which being reduced, will become $3cch + hhh \times 0.5236 = S$; which is one Theorem for finding the Solidity of the Frustum, and may be expressed in Words; thus:

If to three times the Square of the Semidiameter of the Frustum's Base, you add the Square of the Height of the Frustum, and multiply the Sum by the Height of the Frustum, and that Product multiply'd by .5236, the Product will be the solid Content.

But if the Axis of the Sphere, and the Height of the Frustum, be given; then put D = the Axis, h = the Height of the Frustum, and c as before; it will be $D - h = c$, viz. $Dh - hh = cc$. Then will $3Dhh - 2hhh = 3cch + hhh$; consequently, $3Dhh - 2hhh \times 0.5236 = S$, the Frustum's Solidity. Which is another Theorem for finding the Solidity of the Frustum, and may be expressed in Words thus:

From three times the Axis subtract twice the Height of the Frustum; and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this last Product will be the Solidity of the Frustum.



Example. Let ABCD be the Frustum of a Sphere; suppose AB (the Diameter of the Frustum's Base) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By the first Rule.

$$\begin{array}{r}
 8 \\
 8 \\
 \hline
 64 \text{ Square of the Semidiameter AD.} \\
 3 \\
 \hline
 192 \\
 16 \text{ add the Square of CD.} \\
 \hline
 208 \\
 4 \text{ multiply by CD.} \\
 \hline
 832
 \end{array}$$

$$\begin{array}{r}
 .5236 \\
 832 \\
 \hline
 10472 \\
 15708 \\
 41888 \\
 \hline
 435.6352
 \end{array}$$

By the second Rule, thus :

First, by the Rule in Page 113, you will find the Axis of the whole Globe to be 20 Inches.

$$\begin{array}{r}
 20 \text{ Axis} \\
 3 \\
 \hline
 \text{From 60} \\
 \text{Subtr. 8 twice CD.} \\
 \hline
 \text{Rem. 52} \\
 \text{Mult. 16 Sq. of CD.}
 \end{array}$$

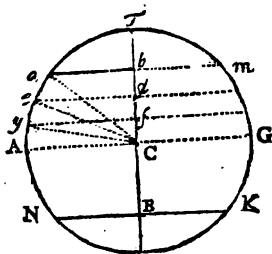
$$\begin{array}{r}
 .5236 \\
 832 \\
 \hline
 10472 \\
 15708 \\
 41888 \\
 \hline
 435.6352
 \end{array}$$

} the solid Content,
 } the same as before.

$$\begin{array}{r}
 312 \\
 52 \\
 \hline
 \text{Prod. 832}
 \end{array}$$

And if it be requir'd to find the middle Part, amNK, usually call'd the middle Zone of a Sphere,

Then, because it is suppos'd that $am = NK$, or (which is all one) that $bC = CB$; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone amNK.



But because that Work is a little troublesome, I will here shew how to raise a Theorem for the doing it,

First, because $AC = yC = eC = aC = TC$, therefore it will be $\square AC - \square Cf = yf$, $\square AC - \square Cd = ed$, $\square AC - \square Cb = ab$, &c.

Here, because $\square AC$, $\square AC$, $\square AC$, &c. are a Series of Equals, and Cb the Number of all the Terms; therefore $\square AC \times Cb =$ the Sum of all that Series, *per* Lemma I.

And $\square Cf$, $\square Cd$, $\square Cb$, &c. being a Series of Squares, whose Roots are in arithmetical Progression, beginning at the Centre, C , viz. o , Cf , Cd , Cb , &c. wherein the greatest Term is $\square Cb$, and the Number of Terms is Cb ; therefore $\square Cb \times \frac{1}{3} Cb =$ the Sum of all the Series, *per* Lemma III.

Consequently, the $\odot AC \times Cb : \odot Cb \times \frac{1}{3} Cb =$ the Sum of all the Series $\odot yf$, $\odot ed$, $\odot ab$, &c. which do constitute the Solidity of the half Zone amAG.

And

Chap. 2. *Mensuration of Solids.* 195

And because $\square AC - \square Cb = \square ab$, therefore
 $\odot AC - \odot ab = \odot Cb$. Consequently $\odot AC \times Cb :$
 $-\odot AC + \odot ab : \times Cb$
 $\frac{3}{= 2\odot AC + \odot ab : \times \frac{1}{3} Cb}$ will
 be Solidity of the half Zone.

Put $D=AG=2AC$, $x=am$, and $H=bB=2Cb$.

Then $\odot AC = .7854 DD$, $\odot ab = .7854 xx$. And
 if we turn the common Factor $.7854$ into a Divisor
 1.27323 , and then take the Triple of that Divisor,
viz. 3.8197 , the Result of the precedent Work will
 produce this following Theorem.

$$\text{Theo. } \left\{ \frac{2DD + xx}{3.8197} : \times H = \right\} \begin{array}{l} \text{the middle Zone} \\ \text{a m N K.} \end{array}$$

Which in Words is thus: To twice the Square of
 the Axis AG , and the Square of the Diameter of the
 Frustrum's Base (am), and divide the Sum by 3.8197 ,
 then multiply the Quotient by the Height or Thick-
 ness of the middle Zone, and the Product will be the
 Solidity of the middle Zone required.

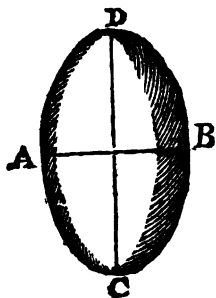
This is so plain and easy, it needs no Example.

§ XII. Of a SPHEROID.

A Spheroid is a Solid resembling an Egg. To find the solid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of the greatest Circle by the Length, and that Product multiply again by .5236; this last Product will be the Solidity of the Spheroid.



Let AB, the Diameter of the greatest Circle, be 33 Inches, and CD (the Length) 55 Inches; the Solidity is requir'd.

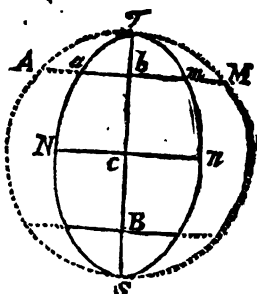
33	59895
33	.5236
<hr/>	<hr/>
99	359370
99	179685
<hr/>	<hr/>
1089	119790
55	299475
<hr/>	<hr/>
5445	31361.0220 the Solidity.
5445	
<hr/>	
59895	

Demon-

Chap. 2. Mensuration of Solids. 197

Demonstration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whose Base is equal to the greatest Circle of the Spheroid, and its Height equal to the Length of the Spheroid.

Suppose the Figure NTnSN in the annex'd Scheme, to represent a Spheroid, form'd by the Rotation of the Semi-Ellipsis TNS, about its transverse Axis TS.



Let $D = TS$, the Length of the Spheroid, and the Axis of its circumscribing Sphere; and $d = Nn$, the Diameter of the greatest Circle of the Spheroid.

Then because, $\square T C : \square N C :: \square A b : \square a b$, by *Se \bar{c} . XV. Step. 3. Page 125.*

Therefore it will be, $DD : dd :: \square A b : \square a b$.

But the Sum of an infinite Series of such Circles as $\odot A b$ (whose Diameters are Chords) do constitute the Solidity of the Sphere. (By *Se \bar{c} . XI.*)

And the Sum of an infinite Series of such Circles as $\odot a b$ (*viz.* whose Diameters are Ordinates of the Ellipsis) do constitute the Solidity of the Spheroid.

Therefore, $DD : dd :: 0.5236 DDD : 0.5236 Ddd =$ the Solidity of the Spheroid. (*Eucl. 5. 12.*)

But $0.5236 Ddd = \frac{2}{3}$ of the Cylinder, whose Diameter is $= d$, and Height $= D$. (By *Se \bar{c} . V.*)

Now, from this Proportion between the Sphere and its inscrib'd Spheroid, it will be very easy to deduce Theorems for finding the solid Content, either of the Frustum or middle Zone of any Spheroid; having the same Height with that of the Sphere; for,

As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid; so is any Part of the Sphere to the like Part of the Spheroid.

198 *Mensuration of Solids.* Part II.

As for Instance : Suppose it was requir'd to find the middle Zone of any Spheroid.

Let $D=TS$, and $d=Nn$, as above ; and $H=bB$, $x=AM$, and $c=am$.

Then $\left\{ \frac{2DD+xx}{3.8197} \right\} \times H =$ the middle Zone of the Sphere. And $0.5236DDD : 0.5236ddD :: \frac{2DD+xx}{3.8197} \times H : \frac{2ddH}{3.8197} + \frac{xxdH}{3.8197DD} =$ the middle Zone of the Spheroid.

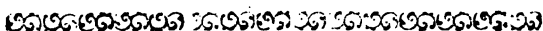
Again, $DD : dd :: xx : cc$. Therefore $\frac{xxdd}{DD} = cc$.

Consequently, $\frac{xxdd}{DD} \times \frac{H}{3.8197} = \frac{cc}{3.8197} \times H$. Which

being taken instead of $\frac{xxddH}{3.8197DD}$, there will arise

this following Theorem $\left\{ \frac{2dd+cc}{3.8197} \right\} \times H =$ the middle Zone of the Spheroid.

Note, That $3.8197 = 1.2732 \times 3$. See Page 102.



§ XIII. Of a Parabolic CONOID.

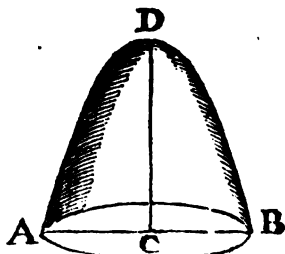
A Parabolic Conoid is something like an half Spheroid, having its Sides somewhat straighter. It is generated by supposing a Semi-parabola turn'd about its Axis. To find the solid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Base by .7854, and multiply that Product by half the Height, that last Product shall be the solid Content.

Let

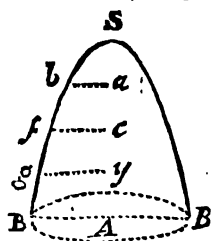
Let ABCD be a Parabolic Conoid, the Diameter of whose Base is 36 Inches, and its Height CD 33 Inches; the Solidity is requir'd.



36	.7854	1017.8784
36	1296	33
216	47124	30536352
108	70686	30536352
1296	15708	2)33589.9872
	7854	
	1017.8784	16794.9936
		1728)16794.9936(9.71 Feet the Content.
	15552	
	12429	
	12096	
	3339	
	1728	
	1611	

Demonstration. The Parabolic Conoid is constituted of an infinite Number of Circles, whose Diameters are the Ordinates of the Parabola. Now according to the Property of every Parabola, it will be, $SA : AB :: AB : \frac{AB^2}{SA} = L$. the *Latus Rectum*.

Then



Then $\begin{cases} S a \times L = \square ba, \\ S c \times L = \square fe, \\ S y \times L = \square gy, \&c. \end{cases}$

Here $S a \times L$, $S c \times L$, $S y \times L$, $\&c.$ are a Series of Terms in Arithmet. Progreff. Therefore $\square ba$, $\square fe$, $\square gy$, $\&c.$ are also a Series of Terms in the same Progreffion, beginning at the Point S, wherein $\square AB$ is the greatest Term, and SA the Number of all the Terms. Therefore $\square AB \times \frac{1}{2} SA =$ the Sum of all the Series. (By Lemma 2.)

Consequently, $\odot AB \times \frac{1}{2} SA =$ the Sum of all the Series of $\odot ba$, $\odot fe$, $\odot gy$, $\&c.$ which do constitute the Solidity of the Conoid.

Put $D = 2 AB$, and $H = SA$.

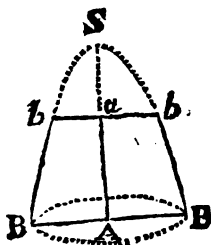
Then $.7854 DD \times \frac{1}{2} H = .3927 DDH$ will be the solid Content of the Conoid; which is just half the Cylinder, whose Base is $= D$, and Height $= H$.

This being rightly understood, it will be easy to raise a Theorem for finding the lower Frustrum of any Parabolic Conoid.

For, supposing $h = aA$, the Height of the Frustrum, and $p = Sa$, the Height of the Part bSb cut off; and $h + p = SA$, the Height of the whole Conoid.

Consequently, $\frac{\odot AB \times h + \odot AB \times p}{2} =$ the Solidity of the whole Conoid.

And $\frac{\odot ba \times p}{2} =$ the Solidity of the Part cut off.



Therefore

Therefore	1	{	$\ominus AB \times h + \ominus AB \times p - \ominus ba \times p$
			$\frac{2}{}$
			The Solidity of the Fruustum.
But	2		$h + p : \square AB :: p : \square ba$
Consequ.	3		$h + p : \ominus AB :: p : \ominus ba$
	4		$\ominus AB \times p = \ominus ba \times ph + \ominus ba \times p$
4 -	5		$\ominus AB \times p = \ominus ba \times h + \ominus ba \times h$
	6	{	$\ominus AB \times h : + \ominus AB \times p : - \ominus ba$
			$\times p = 2 F.$
	7		$\ominus AB \times h = 2 F : - \ominus ba \times h.$
7 +	8		$\ominus AB \times h : + \ominus ba \times h = 2 F.$
	9	{	$\frac{\ominus AB + \ominus ba}{2} : \times h = F, \text{ the Fru-}$
8 ÷ 2			stum's Solidity.

Let $D = 2 AB$, as before, and $d = 2 ba$, the Diameter of the Part cut off; then we shall have this following Theorem.

$0.3927 DD + 0.3927 dd : \times h =$ the Solidity of the Fruustum requir'd: Which in Words is thus:

Multiply the Sum of the Squares of the greater and lesser Diameters by .3927, and that Product by the Height of the Fruustum, the last Product shall be the solid Content.

~~~~~

#### § XIV. *Of a Parabolic SPINDLE.*

**I**F an acute Parabola be suppos'd to be mov'd about its greatest Ordinate, it will form a Solid, call'd a Parabolic Spindle. To find the solid Content, this is

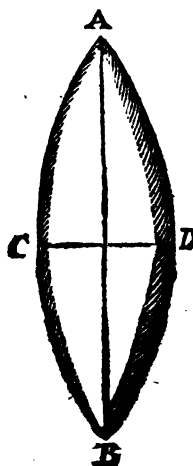
*The*

## The R U L E.

Multiply the Square of the Diameter of its greatest Circle by .41888, (being  $\frac{4}{9}$  of .7854) and that Product by its Length; that laſt Product is the ſolid Content.

Let ABCD be a Parabolic Spindle, whoſe great-eſt Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is requir'd.

$$\begin{array}{r}
 36 = CD \quad .41888 \\
 36 \quad \quad 1296 \\
 \hline
 216 \quad \quad 251328 \\
 108 \quad \quad 376992 \\
 \hline
 1296 \text{ Square } 41888 \\
 \hline
 542.86840 \\
 \quad \quad 99 \\
 \hline
 488581632 \\
 488581632 \\
 \hline
 1728)53743.97952(31.10184 \\
 \quad \quad \quad \dots \\
 \hline
 1903 \\
 1759 \\
 \quad 3179 \\
 \quad 14515 \\
 \quad \quad 6912 \\
 \hline
 \quad \quad \quad \dots
 \end{array}$$



The ſolid Content is 31.10184 Feet.

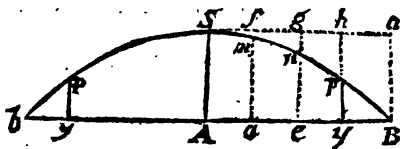
*Demonſtration*

*Demonstration.* A Parabolic Spindle is constituted of an infinite Series of Circles, whose Diameters are all parallel to the Axis of the Parabola, as  $\odot ma$ ,  $\odot ne$ ,  $\odot py$ , &c.

Let us suppose the Line  $Sd$  parallel to  $AB$ , &c. Then it hath already been prov'd, that the Lines  $fm$ ,  $gn$ ,  $hp$ , &c.

are a Series of Squares, whose Roots are in arithmetical Progression, consequently

their Squares, viz.  $\square fm$ ,  $\square gn$ ,  $\square hp$ , &c. will be a Series of Biquadrats, whose Roots will be arithmetical Progression: Which being premis'd, we may proceed thus:



$$\begin{array}{l} \text{First } \left\{ \begin{array}{l} 1 \text{ } \odot \text{ } 2 \text{ } 4 \text{ } | \text{ } 1 \text{ } SA - fm = ma. \\ 2 \text{ } \odot \text{ } 2 \text{ } 5 \text{ } | \text{ } 2 \text{ } SA - gn = ne. \\ 3 \text{ } \odot \text{ } 2 \text{ } 6 \text{ } | \text{ } 3 \text{ } SA - hp = py. \end{array} \right. \\ 1 \text{ } \odot \text{ } 2 \text{ } 4 \text{ } | \text{ } \square SA - 2 \square SA \times fm + \square fm = \square ma. \\ 2 \text{ } \odot \text{ } 2 \text{ } 5 \text{ } | \text{ } \square SA - 2 \square SA \times gn + \square gn = \square ne. \\ 3 \text{ } \odot \text{ } 2 \text{ } 6 \text{ } | \text{ } \square SA - 2 \square SA \times hp + \square hp = \square py, \&c. \end{array}$$

1. In these Equations, the  $\square SA$ ,  $\square SA$ ,  $\square SA$ , being a Series of Equals, and  $AB$  the Number of all the Terms; therefore it will be  $\square SA \times AB =$  the Sum of all the Series, by Lemma I.

2. Because  $fm$ ,  $gn$ ,  $hp$ , &c. are a Series of Squares, wherein  $SA$  is the greatest Term, and  $AB$  the Number of all the Terms;

Therefore  $\frac{2 \square SA \times SA \times AB}{3} = \frac{2 \square SA \times AB}{3}$  will be the Sum of all the Series, by Lemma III.

204 *Mensuration of Solids.* Part II.

3. And the  $\square f m$ ,  $\square g n$ ,  $\square h p$ , &c. will be a Series of Terms in the Ratio of Biquadrats as above.  $\square SA$  being the greatest Term, and  $AB$  the Number of all the Terms. Therefore it will be  $\frac{\square SA \times AB}{5}$

= the Sum of all the Series, by Lemma V.

Whence it follows, that  $\square SA \times AB^2 \frac{\square SA \times AB}{3}$

+  $\frac{\square SA \times AB}{5}$  = the Sum of all the Series of  $\square ma$ ,  $\square ne$ ,  $\square py$ , &c.

That is,  $\frac{8 \square SA \times AB}{15}$  = the Sum of all the Series,

$\square ma$ ,  $\square ne$ ,  $\square py$ , &c. Consequently,  $\frac{8 \odot SA \times AB}{15}$

= the Sum of all the Series of Circles,  $\odot ma$ ,  $\odot ne$ ,  $\odot py$ , &c. which constitute the Solidity of half the Spindle, viz. of  $SA B$ .

Therefore putting  $D = 2 SA$ , and  $H = 2 AB$ , it will be  $0.41888 D D H$  = the Solidity of the whole Parabolic Spindle  $b SB$ , being  $\frac{1}{3}$  of  $0.7854 D D H$ , the Solidity of its circumscribing Cylinder.

From hence we may also raise a Theorem for finding the Fruustum,  $SA py$ , of the last Figure.

For  $\odot SA$  being the greatest Term,  $\odot py$  the least Term, and  $Ay$  the Number of all the Terms or Circles included between  $A$  and  $y$  :

Therefore



Therefore  $\left\{ \begin{array}{l} 1 \square SA - \frac{2 SA \times hp}{3} + \frac{\square hp}{5} : \times Ay = z \\ \text{the Sum of all the Series, } \square SA, \\ \square ma, \square en, \square py, \\ 1 \times 3 \quad 2 \quad 3 \square SA - 2 SA \times hp + \frac{3 \square hp}{5} : \times Ay = 3z \\ z - Ay \quad 3 \quad 3 \square SA - 2 SA \times hp + \frac{3 \square hp}{5} = \frac{3z}{Ay} \\ \text{But} \quad 4 \square SA - 2 SA \times hp = \square py - \square hp, \text{ per St. 6.} \\ 3 - 4 \quad 5 \quad 2 \square SA + \frac{\square 3hp}{5} = \frac{3z}{Ay} - \square py + \square hp \\ 5 + 6 \quad 2 \square SA + \square py - \frac{2}{5} \square hp = \frac{3z}{Ay} \end{array} \right.$

Conseq.  $7 \quad 2 \square SA + \square py - \frac{2}{5} \square hp : \times \frac{1}{3} Ay = z$   
 the Sum of all the Series of  $\square SA, \square ma, \square en, \square py$ ,  
 which do constitute the Solidity of the Frustum  $SApy$ .  
 Therefore, putting  $D = 2 SA$ , as before,  $C = 2 py$ ,  
 $x = 2hp$ , and  $H = Ay$ ; it will be  $1.5708 DD + .7854$   
 $CC - .31416 xx : \times \frac{1}{3} H =$  the Frustum  $SApy$ . And  
 if we make  $L = 2H$ , then  $1.5708 DD + .7854 CC -$   
 $.31416 xx : \times \frac{1}{3} L =$  the Double of that Frustum, be-  
 ing the middle Zone. Which in Words is thus;

Multiply the Square of the greatest Diameter by  
 1.5708, and multiply the Square of the lesser Dia-  
 meter by .7854, and multiply the Square of the  
 Difference of the Diameters by .31416; from the  
 Sum of the two former Products subtract the latter  
 Product, and multiply the Remainder by one third  
 Part of the Length, and that Product will be the Soli-  
 dity of the middle Zone required.

EXAMPLES.  
 1. A Frustum of a Cone, the greatest Diameter of which is 10, the  
 least 6, and the Length 12, required the Solidity of the middle  
 Zone.  
 2. A Frustum of a Cone, the greatest Diameter of which is 12, the  
 least 8, and the Length 10, required the Solidity of the middle  
 Zone.  
 3. A Frustum of a Cone, the greatest Diameter of which is 14, the  
 least 10, and the Length 15, required the Solidity of the middle  
 Zone.



### CHAP. III.

*Of the Measuring the Works of the several Artificers relating to Building; and what Methods and Customs are observed therein.*



#### § I. Of CARPENTERS Work.

**T**HE Carpenters Work which are measurable, are Flooring, Partitioning, and Roofing; all which are measured by the Square of 10 Feet long, and 10 Feet broad; so that one Square contains 100 square Feet.

##### 1. Of Flooring.

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad; How many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, &c. which divide by 100 (this is done by cutting from the Product two Figures towards the Right-hand with a Dash of the Pen)

# Chap. 3. CARPENTERS Work.

203

Pen) ; and the remaining Figures are the Quotient and the Figures cut off are Feet: Thus, 1631 divided by 100, by cutting off 31 from the Right-hand thereof, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work, both by Decimals, and also by Feet and Inches.

|           |           |
|-----------|-----------|
| 57 25     | F. I.     |
| 28.5      | 57 3      |
| <hr/>     | 28 6      |
| 28625     | <hr/>     |
| 45800     | 456       |
| 11450     | 114       |
| <hr/>     | 28 7 6    |
| 16 31.625 | 7 0 0     |
|           | <hr/>     |
|           | 16 31 7 6 |

*Facit* 16 Squares and 31 Feet.

*Note*, That .5 is the Decimal for half of any thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; so in the last Example, .25 is the Decimal of 3 Inches, because 3 Inches is a Quarter of a Foot; and .5 is the Decimal of 6 Inches, because 6 Inches is half a Foot.

*Example 2.* Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad, How many Squares are contain'd in that Floor?

|            |           |
|------------|-----------|
| 47.75      | F. I.     |
| 53.5       | 53 6      |
| <hr/>      | 47 9      |
| 23875      | <hr/>     |
| 14325      | 371       |
| 23875      | 212       |
| <hr/>      | 26 9      |
| 25' 54.625 | 13 4 6    |
|            | 23 6      |
|            | <hr/>     |
|            | 25 54 7 6 |

*Facit* 25 Squares and 54 Feet.

T 2

By

## By Scale and Compasses.

In the first Example, extend the Compasses from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the second Example, extend the Compasses from 1 to 47.5, that Extent will reach from 53.5 to 25 Squares and above an Half.

## 2. Of Partitioning.

*Example 1.* If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiply'd together, the Product is 1010 625; which divide by 100, (as before is shew'd) and the Answer is 10 Squares 10 Feet; the Inches or Parts, in these Cases, are of no Value.

|          |          |
|----------|----------|
| 12.25    | F. 1.    |
| 82.5     | 82 6     |
| <hr/>    | <hr/>    |
| 6125     | 12 3     |
| 2450     | 990 0    |
| 9800     | 20 7 6   |
| <hr/>    | <hr/>    |
| 1010.625 | 1010 7 6 |

*Facit* 10 Squares 10 Feet.

*Example 2.* If a Partition between Rooms be in Length 91 Feet 9 Inches, and in Breadth 11 Feet 3 Inches; How many Squares are contain'd therein?

The Length and Breadth being multiply'd together, the Product is 1032 Feet; which divided by 100, the Answer will be 10 Squares and 32 Feet.

$$\begin{array}{r}
 91.75 \\
 11.25 \\
 \hline
 45875 \\
 18350 \\
 9175 \\
 9175 \\
 \hline
 1032.1875
 \end{array}$$

$$\begin{array}{r}
 \text{F, I.} \\
 91 \quad 9 \\
 11 \quad 3 \\
 \hline
 1009 \quad 3 \\
 22 \quad 11 \quad 3 \\
 \hline
 10132 \quad 2 \quad 3
 \end{array}$$

### 3. Of Roofing.

It is a Rule amongst Workmen, that the Flat of any House, and half the Flat thereof, taken within the Walls, is equal to the Measure of the Roof of the same House; but this is when the Roof is true pitched: For if the Roof be more flat or steep than the true Pitch, it will measure to more or less accordingly.

*Example 1.* If a House within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; How many Squares of Roofing will cover that House?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet; which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Answer is 12 Squares and 18 Feet.

|              |                                  |
|--------------|----------------------------------|
| 18.25        | F. I.                            |
| 44 5         | 44 6                             |
| <hr/>        | <hr/>                            |
| 9125         | 18 3                             |
| 7300         | <hr/>                            |
| 7300         | 352                              |
| <hr/>        | 44                               |
| Flat 812.125 | 11 1 6                           |
| Half 406     | 9 0 0                            |
| <hr/>        | <hr/>                            |
| 12.18        | The Flat 812 1 6                 |
|              | The Half 406                     |
|              | <hr/>                            |
|              | Sum 12.18                        |
|              | <i>Facit</i> 12 Squares 18 Feet. |

*By Scale and Compasses.*

In the first Example of Partitioning, extend the Compasses from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and one Tenth.

In the second Example, extend the Compasses from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little less than a third Part.

In the Example of Roofing, extend the Compasses from 1 to 18.25, that Extent will reach from 44.5 to 812 the Flat; to which add the Half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, that are measured by the Foot, running Measure, that is, by the Number of Feet in Length only; as Cornices, Doors and Cases, Window-frames, Gutterin, Lintels, Sommers, Skirt-boards, &c.

*Note 1.* In the measuring of Flooring, after you have measured the whole Floor, you must deduct out of it the Well-holes for the Stairs and Chimneys; and in Partitioning, for the Doors, Windows, &c. except (by Agreement) they are to be included.

## Chap. 3. BRICKLAYERS Work.

217

*Note 2.* In measuring of Roofing, seldom any Reductions are made for the Holes for the Chimney-shafts, the Vacancies for Lutheren-lights and Skylights ; for they are more Trouble to the Workman, than the Stuff which would cover them is worth.



### § II. Of BRICKLAYERS Work.

**T**HE principal is Tiling, Walling, and Chimney-work.

#### 1. Of Tiling.

Tiling is measured by the Square of 100 Feet, as Flooring, Partitioning and Roofing were in the Carpenters Work ; so that between the Roofing and Tiling, the Difference will not be much ; yet the Tiling will be the most ; for the Bricklayers sometimes will require to have double Measure for Hips and Vallies. When Gutters are allowed double Measure, the way is to measure the Length along the Ridge-tile, and by that means the Measure of the Gutters becomes double ; it is usual also to allow double Measure at the Eaves, so much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

*Example 1.* There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet ; I demand how many Squares of Tiling are contained therein ?

|       |      |          |
|-------|------|----------|
| F.    | I.   |          |
| 37    | 3    | 37.25    |
| 45    | 0    | 45       |
| <hr/> |      | <hr/>    |
| 185   |      | 18625    |
| 148   |      | 14900    |
| 11    | 3    | <hr/>    |
| <hr/> |      | 16,762.5 |
| 16    | 76 3 |          |

Answer, 16 Squares 76 Feet.

*Example 2.* There is a Roof covered with Tiles, whose Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tiling are in the Roof?

|       |        |           |
|-------|--------|-----------|
| F.    | I.     |           |
| 43    | 6      | 35.75     |
| 35    | 9      | 43.5      |
| <hr/> |        | <hr/>     |
| 215   |        | 17875     |
| 129   |        | 10725     |
| 21    | 9      | <hr/>     |
| 30    | 10 6   | 14300     |
| 17    | 6      | <hr/>     |
| <hr/> |        | 15,55.125 |
| 15    | 55 1 6 |           |

Here the Length and Depth being multiply'd together, the Product is 1555 Feet; which divided by 100, (as before is taught) the Answer is 15 Squares and 55 Feet.

### By Scale and Compasses.

In the first Example, extend the Compasses from 1 to 37.25, that Extent will reach from 45 to 16 Squares and a little above three Quarters of a Square.



In the second Example, extend the Compasses from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that is, a little above a Half-square.

## 2. Of Walling.

Bricklayers commonly measure their Work by the Rod square of 16 Feet and a half; so that one Rod in Length and 1 in Breadth, contain 272.25 square Feet; for 16.5, multiply'd in itself, produces 272.25 square Feet. But in some Places the Custom is to allow 18 Feet to the Rod; that is, 324 square Feet. And in some Places the usual Way is, to measure by the Rod of 21 Feet long and 3 Feet high, that is, 63 square Feet; and here they never regard the Thickness of the Wall, but the usual Way is to moderate the Price according to the Thickness.

When you measure a Piece of Brick-work, the first thing is to inquire by which of those Ways it must be measured; then, having multiply'd the Length and Breadth in Feet together, divide the Product by the proper Divisor, either for Rods or Roods, and the Quotient is square Rods, or square Roods, accordingly.

But commonly Brick-walls, that are measured by the Rod, are to be reduced to a Standard-Thickness, *viz.* of a Brick and a half thick (if it be not agreed on to the contrary); and to reduce a Wall to Standard-thickness, this is

## The R U L E.

Multiply the Number of superficial Feet that are found to be contain'd in any Wall, by the Number of Half-bricks which that Wall is in Thickness; one third Part of that Product shall be the Content thereof in Feet, reduced to the Standard-thickness of one Brick and a half.

*Example*

*Example 1.* If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick; How many Rods of Brick-work are contained therein when reduced to the Standard?

19.25 Height.  
72.4 Length.

9625  
3850  
13475

1395.625  
11

3)1535 875

272.25) 5117.291 (18 Rods.

239475

68.06)21679 (3 Quarters of a Rod,

12.61

Answer, 18 Rods 3 Quarters 12 Feet.

$$\begin{array}{r}
 \begin{array}{cc}
 \text{F.} & \text{I.} \\
 72 & 6 \\
 19 & 3 \\
 \hline
 648 & \\
 72 & \\
 18 & 1 \quad 6 \\
 9 & 6 \quad 0 \\
 \hline
 1395 & 7 \quad 6 \\
 11 & \\
 \hline
 3)15345 & \\
 \hline
 272)5115(18 \text{ Rods.} & \\
 \hline
 2395 & \\
 \hline
 68)219(3 \text{ Quarters of a Rod.} & \\
 \hline
 15 &
 \end{array}
 \end{array}$$

*Note,* That 68.06 is one fourth-Part of 272.25.

*Note also,* That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as is done in the second Way of the last Example; so the Answer, by that second Way, is 18 Rods 3 Quarters and 15 Feet, more by about  $2\frac{1}{2}$  Feet than by the first Way, where it is done decimally; a Thing very insignificant.

*Example*

*Example 2.* If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and two Bricks and a half thick; I demand how many Rods of Brick-work are contained therein, when reduced to Standard-Thickness?

$$\begin{array}{r} 245.75 \\ 16.5 \\ \hline \end{array}$$

$$\begin{array}{r} 122875 \\ 147450 \\ 24575 \\ \hline \end{array}$$

$$\begin{array}{r} 4054.875 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \overline{)20270} \\ \hline \end{array}$$

$$272,6756(24 \text{ Rods.}$$

$$\begin{array}{r} 1316 \\ 68 \overline{)228} 3 \text{ Quarters of a Rod.} \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \text{Answer, } 24 \text{ Rods } 3 \text{ Quarters } 24 \text{ Feet.} \end{array}$$

F. I.

$$\begin{array}{r} 245 \quad 9 \\ 16 \quad 6 \\ \hline \end{array}$$

$$1470$$

$$245$$

$$122 \quad 10 \quad 6$$

$$12 \quad 0 \quad 0$$

$$4054 \quad 10 \quad 6 \text{ Answer in Feet.}$$

Before I shew how to work the two last Examples by Scale and Compasses, I will shew how to find proper Divisors to facilitate the Operation; because it would be too intricate and tedious to perform by Scale and Compasses, according to the Rule above taught.

*To find proper Divisors.*

Divide 3 (the Number of Half-bricks in  $1\frac{1}{2}$ ) by the Number of Half-bricks in the Thickness, the Quotient will be a Divisor, which will give the Answer in Feet. But if you would have a Divisor to bring the Answer in Rods at once, then multiply 272.25 by the Divisor found for Feet, and the Product will be a Divisor which will give the Answer in Rods.

*Example.* Let it be required to find a Divisor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6, (the Half-bricks in the Thickness) and the Quotient is .5, which is a Divisor that will give the Answer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Divisor, which will give the Answer in Rods; that is, as 136.125 is to the Length of the Wall, so is the Height to the Content in Rods. Or, as .5 is to the Length, so is the Height to the Content in Feet.

After the same manner you may find Divisors for any other Thickness, which you will find to be as express'd in the following little Table.

| The Thickness<br>of the Wall. | Divisors<br>for the<br>Answer<br>in Feet. | Divisors<br>for bringing<br>the Answer<br>in Rods. |
|-------------------------------|-------------------------------------------|----------------------------------------------------|
| 1 Brick thick                 | 1.5                                       | 408.375                                            |
| 1 $\frac{1}{2}$ Brick thick   | 1.                                        | 272.25                                             |
| 2 Bricks thick                | .75                                       | 204.1875                                           |
| 2 $\frac{1}{2}$ Bricks thick  | .6                                        | 163.35                                             |
| 3 Bricks thick                | .5                                        | 136.125                                            |
| 3 $\frac{1}{2}$ Bricks thick  | .4285                                     | 116.659                                            |
| 4 Bricks thick                | .375                                      | 102.0937                                           |

Let the second Example, foregoing, be wrought by Scale and Compasses, where the Length is 245.75, the Height 16.5, and the Thickness 2  $\frac{1}{2}$  Bricks.

Extend the Compasses from 163.35, (the tabular Number against 2  $\frac{1}{2}$  Bricks) to 245.75; that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3  $\frac{1}{2}$  Bricks thick; How many Rods are contained therein?

Extend the Compasses from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13; that is 12 Rods and a little above half a Quarter.

It will be very proper and commodious, for such as have frequent Occasion to measure Brick-work, to have in the Line of Numbers little Brass Centre-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickness of the Wall.

If

# Chap. 3. BRICKLAYERS *Work.* 219

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Rods are contain'd therein?

|                  | F.   | I.   |
|------------------|------|------|
| 104.75           | 104  | 9    |
| 17.25            | 17   | 3    |
| 52375            | 728  |      |
| 20950            | 104  |      |
| 73325            | 26   | 2 3  |
| 10475            | 12   | 9 0  |
| 63)1806.9375(28. | 1806 | 11 3 |
| 126              |      |      |
| 546              |      |      |
| 504              |      |      |
| 42               |      |      |

Answer, 28 Rods 42 Feet.

*Note,* That such as dig Cellars, do many times do them by the Floor, 16 Feet square, and a Foot deep, being a Floor of Earth, that is, 324 solid Feet.

## 3. Of Chimnies.

If you are to measure a Chimney standing alone by itself, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickness must be the same as the Jambs are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, because of the Gatherings of the Breast and Wings, to make room for the Hearth in the next Story.

If the Chimney-back be a Party-wall, and the Wall be measured by itself, then you must measure the Depth of the two Jambs, and the Length of the Breast for a Length, and the Height of the Story the Breadth, at the same Thickness your Jambs were of.

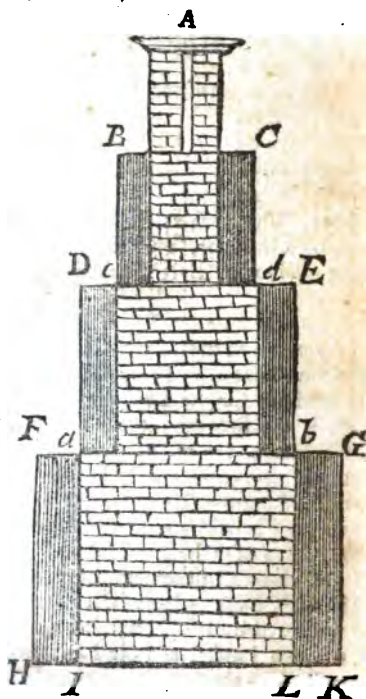
When you measure Chimney-shafts, girt them with a Line round about the least Place of them, for

the Length, and the Height shall be your Breadth : And if they be four Inch Work, then you must set down your Thickness at one Brick-work ; but if they be wrought 9 Inches thick, (as sometimes they are, when they stand high and alone above the Roof) then you must account your Thickness  $1\frac{1}{2}$  Brick, in Consideration of Withs and Pargetting, and Trouble in Scaffolding.

It is customary, in most Places, to allow double Measure for Chimneys.

*Example.* Suppose this Figure, ABCDEFGHK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be measur'd according to double Measure.

First, I begin with the Breast-wall I L, and the two Angles L K and H I, which together are 18 Feet 9 Inches ; then take the Height of the Square H F, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.



For



# Chap. 3. BRICKLAYERS Work 221

For the Square DaEb, the Length of the Breast-wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet ; which multiply'd together, make 130 Feet 6 Inches, for the Content of the Part Da Eb.

Then the Height of the next Square 7 Feet, and the Length of the Breast-wall and two Angles is 10 Feet 3 Inches ; which multiply'd together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compaſs of the Chimney-shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches ; which multiply'd together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnel, is 12 Feet, and its Wideneſs 1 Foot 3 Inches ; which multiply'd together, make 15 Feet, the Content thereof.

The Work.

|       |       |
|-------|-------|
| F. I. | 18.75 |
| 18 9  | 12.5  |
| 12 6  |       |
| <hr/> |       |
| 225 0 | 9375  |
| 9 4 6 | 3750  |
|       | 1875  |
| <hr/> |       |

FGHK 234 4 6

FGHK 234.375

F. I.

|       |      |
|-------|------|
| 14 6  | 14.5 |
| 9 0   | 9    |
| <hr/> |      |

DaEb 130 6

DaEb 130.5

F. I.

|       |       |
|-------|-------|
| 10 3  | 10.25 |
| 7 0   | 7     |
| <hr/> |       |

BcCd 71 9

BcCd 71.75  
F. I.

U 3

|                  |                  |
|------------------|------------------|
| F. I.            | 13.75            |
| 13 9             | 6.5              |
| 6 6              |                  |
| <hr/>            |                  |
| 82 6             | 6875             |
| 6 10 6           | 8250             |
| <hr/>            |                  |
| The Shaft 89 4 6 | The Shaft 89.375 |

|                 |                  |
|-----------------|------------------|
| F. I.           | 1.25             |
| 1 3             | 12               |
| 12 0            |                  |
| <hr/>           |                  |
| The Fetter 15 0 | The Fetter 15.00 |

|                     |                     |
|---------------------|---------------------|
|                     | F. I. P.            |
| 272)1082(3 Rods     | FGHK 234 4 6        |
|                     | DaEb 130 6 0        |
| 68) 266(3 Quarters, | BcCd 71 9 0         |
|                     | The Shaft 89 4 6    |
| Rem. 62 Feet.       | The Fetter 15 0 0   |
|                     | <hr/>               |
|                     | The Sum 541 0 0     |
|                     | <hr/>               |
|                     | The Double 1082 0 0 |

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or customary Measure; which Feet must be reduc'd to Rods, as was shew'd before.

So the Feet in the foregoing Example being reduc'd to Rods, (the Thickness being supposed  $1\frac{1}{3}$  Bricks) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all the Measure that can be allow'd, when the Chimney stands in a Gavel or Side-wall; in which Case the Back of the Chimney (here not measur'd) is accounted as Part of the Gavel; but if the Chimneys stand by themselves, as all Stacks of Chimneys

## Chap. 3. PLASTERERS *Work.* 223

in great Buildings do, in such Case it is all Chimney-work, and therefore ought to be measured double on all Sides.



### § III. Of PLASTERERS *Work.*

**T**HE Plasterers Works are principally of two Kinds, namely, 1. Works lath'd or plaster'd, which they call Cieling. 2. Works render'd; which is of two Kinds, *viz.* upon Brick-walls, or between Quarters, in the Partitions between Rooms; all which are measured by the Yard square, or Square of 3 Feet, which is 9 Feet.

#### 1. Of Cieling.

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad, how many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

|                                                                                                                                                             |                                                                                                                                                                                                          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| F. I.<br>59 9<br>24 6<br><hr style="width: 100px; margin: 5px 0;"/> 236<br>118<br>29 10 6<br>18 0 0<br><hr style="width: 100px; margin: 5px 0;"/> 1463 10 6 | 59.75<br>24.5<br><hr style="width: 100px; margin: 5px 0;"/> 29875<br>23900<br>11950<br><hr style="width: 100px; margin: 5px 0;"/> 9)1463.875<br><hr style="width: 100px; margin: 5px 0;"/> Answer 162.65 |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

*By Scale and Compasses.*

Extend the Compasses from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 16.25 Yards.

*2. Of Rendering.*

*Example.* If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high, How many Yards are in those Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Answer.

| F.     | I.    |            |
|--------|-------|------------|
| 141    | 6     | 141.5      |
| 11     | 3     | 11.25      |
| <hr/>  |       | <hr/>      |
| 1556   | 6     | 7075       |
| 35     | 4 6   | 2830       |
| <hr/>  |       | <hr/>      |
| 9)1591 | 10 6  | 1415       |
| <hr/>  |       | <hr/>      |
| Answer | 176 7 | 1415       |
|        |       | <hr/>      |
|        |       | 9)1591.875 |
|        |       | <hr/>      |
|        |       | 176.87     |

Answer, 176.87 Yards.

Extend the Compasses from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

*Note 1.* If there be any Doors, Windows, or the like, in your Partitioning, you must make Deductions for them.

*Note 2.* When you measure Rendering upon Brick-walls, you are to make no Deductions; but when you measure Rendering between Quarters, you may very well deduct one-fifth Part for the Quarters, Braces, and Entertices.

*Note*

*Note 3.* That Whiting and Colouring are both measured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deduct one-fifth Part, so in Whiting and Colouring you must add one-fourth, or one-fifth Part at least.



#### § IV. Of JOYNEERS *Work*.

**J**oyners do measure their Work by the Yard square; but in taking their Dimensions, they differ from some others; for they have a Custom, and say, *We ought to measure where our Plane touches*: Wherefore, in taking the Height of any Room, where there is a Cornice about, and swelling Panels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to measure much higher than it is: Then for measuring about the Room, they only take it as it is upon the Floor.

*Example 1.* If a Room of Wainscot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in Compass, How many Yards doth that Room contain?

Multiply the Compass by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Answer.

| F.     | I.  |             |
|--------|-----|-------------|
| 126    | 3   | 126.25      |
| 15     | 9   | 15.75       |
| <hr/>  |     | <hr/>       |
| 630    |     | 63125       |
| 126    |     | 88375       |
| 63     | 1 6 | 63125       |
| 31     | 6 9 | 12625       |
| 3      | 9 0 | <hr/>       |
| <hr/>  |     | 9)1988.4375 |
| 9)1988 | 5 3 | <hr/>       |
| <hr/>  |     | 220 8       |

Answer 220 8

*Facit* 220 Yards 8 Feet.

*Example 2.* If a Room of Wainscot be 16 Feet 3 Inches high, (being girt over the Mouldings) and the Compass of the Room 137 Feet 6 Inches, How many Yards are contain'd therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

| F.     | I.  |            |
|--------|-----|------------|
| 137    | 6   | 137.5      |
| 16     | 3   | 16.25      |
| <hr/>  |     | <hr/>      |
| 830    |     | 6875       |
| 137    |     | 2750       |
| 34     | 4 6 | 8250       |
| <hr/>  |     | 1375       |
| 9)2234 | 4 6 | <hr/>      |
| <hr/>  |     | 9)2234.375 |
| 248    | 2 0 | <hr/>      |
| <hr/>  |     | 248.2      |

*Facit* 248 Yards 2 Feet.

By

*By Scale and Compasses.*

For the first Example, extend the Compasses from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the second Example, extend the Compasses from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards and about a Quarter.

In Joyners Work there is another Thing to be observ'd, that is, in the measuring of Doors, Window-shutters, and all such Work as is wrought on both Sides, they are paid for Work and Half-work; so that in measuring all such Work, you must first find the Content, as before, and take half that Content, and add to it; so shall the Sum be the Content at Work and half.

*Example.* If the Window-shutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high, How many Yards are contain'd therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the Half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

| F. I.                                                                                                                                 |          |
|---------------------------------------------------------------------------------------------------------------------------------------|----------|
| 69 9                                                                                                                                  | 69.75    |
| 6 3                                                                                                                                   | 6.25     |
| 418 6                                                                                                                                 | 34875    |
| 17 5 3                                                                                                                                | 13950    |
| 435 11 3                                                                                                                              | 41850    |
| 217 11 7                                                                                                                              | 435.9375 |
| 9)653 10 10                                                                                                                           | 217.9687 |
| 72 5                                                                                                                                  | 653.9062 |
| <div style="display: flex; justify-content: space-between;"> <span><i>Facit</i> 72 Yards 5 Feet.</span> <span><i>By</i></span> </div> |          |

*By Scale and Compasses.*

Extend the Compasses from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.02; which added together, make 72.6 Yards, the Content at Work and half.

*Note,* That you must make Deductions for all Window-lights; but you must measure the Window-boards, Sopheta-boards, and Cheeks, by themselves.



## § V. Of PAINTERS Work.

**T**HE taking the Dimensions of Painters Work is the same as that of Joiners, by girting over the Mouldings and swelling Panels, in taking the Height; and it is but Reason that they should be paid for that on which their Time and Colour are both expended. The Dimensions thus taken, the casting up, and reducing Feet into Yards, is altogether the same as the Joiners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice-colour'd over. Only take Notice, that Window-lights, Window-bars, Casements, and sash-like Things, they do at so much *per Piece*.

*Example.* If a Room be painted, whose Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compass of the Room 97 Feet 9 Inches, How many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 0 Inches 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.



| F.    | I.        |                 |
|-------|-----------|-----------------|
| 97    | 9         | 97.75           |
| 16    | 6         | 16.5            |
| <hr/> |           | <hr/>           |
| 584   |           | 48875           |
| 98    |           | 58650           |
| 48    | 10 6      | 9775            |
| <hr/> |           | <hr/>           |
| 9)    | 1612 10 6 | 9) 1612.875(179 |
| <hr/> |           | <hr/>           |
| 179   | 1         |                 |

*Facit* 179 Yards 1 Foot.

*By Scale and Compasses.*

Extend the Compasses from 9 to 16 5, that Extent will reach from 97.75 to 179.2 Yards.



§ VI. Of GLASIERS Work.

**G**lasiers do measure their Work by the Foot Square; so that the Length and Breadth of a Pane of Glas in Feet, being multiply'd into each other, produceth the Content.

*Note,* That Glasiers do usually take their Dimensions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part subdivided into 12, &c.

*Example 1.* If a Pane of Glas be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad; How many Feet of Glas are in that Pane?

The Decimal of  $\left\{ \begin{array}{l} 8 \text{ Inches } \frac{3}{4} \\ 4 \text{ Inches } \frac{1}{4} \end{array} \right\}$  is  $\left\{ \begin{array}{l} .729 \\ .354 \end{array} \right\}$

X

F.

| F. I. P.   |          |
|------------|----------|
| 4 8 9      | 4.729    |
| 1 4 3      | 1.354    |
| <hr/>      | <hr/>    |
| 4 8 9      | 18916    |
| 1 6 11 0   | 23645    |
| 1 2 2 3    | 14187    |
| <hr/>      | <hr/>    |
| 6 4 10 2 3 | 4729     |
|            | <hr/>    |
|            | 6.403066 |

Answer, 6 Feet 4 Inches.

*By Scale and Compasses:*

Extend the Compasses from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

*Example 2.* If there be 8 Panes of Glafs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad; How many Feet of Glafs are contained in the said 8 Panes?

The Decimal of  $\left\{ \begin{array}{l} 7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4} \end{array} \right\}$  is  $\left\{ \begin{array}{l} .646 \\ .437 \end{array} \right\}$

| F. I. P.   |           |
|------------|-----------|
| 4 7 9      | 4.646     |
| 1 5 3      | 1.437     |
| <hr/>      | <hr/>     |
| 4 7 9      | 32522     |
| 1 11 2 9   | 13938     |
| 1 1 11 3   | 18584     |
| <hr/>      | <hr/>     |
| 6 8 1 8 3  | 4646      |
| 8          | <hr/>     |
| <hr/>      | <hr/>     |
| 53 5 1 6 0 | 6.676302  |
|            | 8         |
|            | <hr/>     |
|            | 53.410416 |

Facit 53 Feet 5 Inches.

*By Scale and Compasses.*

Extend the Compasses from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compasses from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

*Example 3.* If there be 16 Panes of Glafs, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad; How many Feet of Glafs are contain'd therein?

| F.    | I. | P.     |          |
|-------|----|--------|----------|
| 4     | 5  | 6      | 4.458    |
| 1     | 4  | 9      | 1.395    |
| <hr/> |    |        | <hr/>    |
| 4     | 5  | 6      | 22290    |
| 1     | 5  | 10 0   | 40122    |
|       | 3  | 4 1 6  | 13374    |
| <hr/> |    |        | <hr/>    |
| 6     | 2  | 8 1 6  | 4458     |
| <hr/> |    |        | <hr/>    |
|       |    | 4      | 6.218910 |
| <hr/> |    |        | <hr/>    |
| 24    | 10 | 8 6 0  | 4        |
| <hr/> |    |        | <hr/>    |
|       |    | 4      | 24.87564 |
| <hr/> |    |        | <hr/>    |
| 99    | 6  | 10 0 0 | 4        |
| <hr/> |    |        | <hr/>    |
|       |    |        | 99.50256 |

*Facit* 99 Feet 6 Inches.

*Note,* That instead of multiplying by 16, I have multiply'd by 4 twice, because 4 times 4 is 16.

*By Scale and Compasses.*

Extend the Compasses from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compasses from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

*Note,* That when Windows have Half-rounds at the Top, they measure them at the full Height, as if they were square. Also round or oval Windows are mea-

X 2

fur'd

fur'd at the full Length and Breadth of their Diameters. Likewise Crotchet-windows in Stone-work are all measur'd by their full Squares. And there is Reason for so doing; for the Trouble in taking their Dimensions to work by, the Waste of Glafs in working, and the Time expended in setting up, is far more than the Glafs can be valued at.



### § VII. Of MASONS Work.

**M**ASONS do measure their Work sometimes by the Foot solid, sometimes by the Foot superficial; and in some Places they measure their Walling by the Rood, that is, 21 Feet long, and 3 Feet high, which is 63 square Feet. Examples of each are as follow.

*Example 1.* If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick; How many solid Feet are contain'd in that Wall?

F. I.

$$\begin{array}{r}
 97 \ 5 \\
 18 \ 3 \\
 \hline
 776 \\
 97 \\
 24 \ 4 \ 3 \\
 6 \ 0 \ 0 \\
 1 \ 6 \ 0 \\
 \hline
 1777 \ 10 \ 3 \\
 2 \ 3 \\
 \hline
 3555 \ 8 \ 6 \\
 444 \ 5 \ 6 \ 9 \\
 \hline
 4000 \ 2 \ 0 \ 9
 \end{array}$$

$$\begin{array}{r}
 97.417 \\
 18.25 \\
 \hline
 487085 \\
 194834 \\
 779336 \\
 97417 \\
 \hline
 1777.86025 \\
 2.25 \\
 \hline
 888930125 \\
 355572050 \\
 355572050 \\
 \hline
 4000.1855625
 \end{array}$$

Multi-

### Chap. 3. MASONS Work. 233.

Multiply the Length, Height, and Thickness together, and the last Product is 4000 Feet 2 Inches, the solid Feet contain'd in the Wall.

#### *By Scale and Compasses.*

Extend the Compasses from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the solid Content.

*Example 2.* If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high; How many Feet superficial are contain'd therein?

| P.    | I.   |          |
|-------|------|----------|
| 107   | 9    | 107.75   |
| 20    | 6    | 20.5     |
| <hr/> |      | <hr/>    |
| 2155  | 0    | 53875    |
| 53    | 10 6 | 215500   |
| <hr/> |      | <hr/>    |
| 2208  | 10 6 | 2208.875 |

*Facit* 2208 Feet 10 Inches.

#### *By Scale and Compasses.*

Extend the Compasses from 1 to 107.75, that Extent will reach from 20.6 to 2208.875, the superficial Feet.

X 3

*Example*

234 *The Mensuration of* Part II.

*Example 3.* If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high ; How many Roods are contain'd therein ?

| F. I.    | 112.25         |
|----------|----------------|
| 112 3    | 16.5           |
| 16 6     | <hr/>          |
| <hr/>    | 56125          |
| 676 0    | 67350          |
| 112      | 11225          |
| 56 1 6   | <hr/>          |
| <hr/>    | 63)1852.125(29 |
| 1852 1 6 | <hr/>          |
|          | 592            |
|          | <hr/>          |
|          | 25             |

*Facit* 29 Roods 25 Feet.

*By Scale and Compaffes.*

Extend the Compaffes from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.





## CHAP. IV.

*The Measuring of BOARD and  
TIMBER.*



## § I. Of BOARD MEASURE.

**T**O measure a Board, is no other but to measure a long Square.

*Example 1.* If a Board be 16 Inches broad, and 13 Feet long; How many Feet are contain'd therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the same as before.

$$\begin{array}{r}
 12 : 13 :: 16 \\
 \underline{13} \\
 48 \\
 \underline{16} \\
 12 \overline{) 208} \\
 \underline{\phantom{00}17} 1 \frac{4}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{Or, } 144 : 156 :: 16 \\
 \underline{16} \\
 936 \\
 \underline{156} \\
 144 \overline{) 2496} (17 \frac{4}{2} \\
 \underline{1056} \\
 48
 \end{array}$$

### By Scale and Compasses.

Extend the Compasses from 12 to 13, that Extent will reach from 16 to  $17 \frac{4}{2}$  Feet, the Content.

Or, extend from 144 to 156 (the Length in Inches) that Extent will reach from 16 to  $17 \frac{4}{2}$  Feet, the Content.

*Example 2.* If a Board be 19 Inches broad; how many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and so many Inches in Length, if a Board be 19 Inches broad, will make a Foot,

$$\begin{array}{l}
 \text{Inch. Inch. Inch. Inch.} \\
 19 : 144 :: 1 : 7.58 \text{ fere.}
 \end{array}$$

Extend



Extend the Compasses from 19 to 144, that Extent will reach from 1 to 7 $\frac{5}{8}$ ; that is, 7 Inches and something more than a half. So, if a Board be 19 Inches broad, if you take 7 Inches, and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains; or you may cut off from that Board any Number of Feet desir'd.

For this Purpose there is a Line upon most ordinary Joint-rule, with a little Table plac'd upon the End of all such Numbers as exceed the Length of the Rule, as in this little Table annex'd.

|    |   |   |   |   |   |                 |   |
|----|---|---|---|---|---|-----------------|---|
| 0  | 0 | 0 | 0 | 5 | 0 | 8 $\frac{1}{2}$ | 6 |
| 12 | 6 | 4 | 3 | 2 | 2 | 1               | 1 |
| 1  | 2 | 3 | 4 | 5 | 6 | 7               | 8 |

Here you see, if the Breadth be one Inch, the Length must be 12 Feet; if two Inches, the Length is 6 Feet; if 5 Inches broad, the Length is 2 Feet 5 Inches, &c.

The rest of the Lengths are express'd in the Line, thus: If the Breadth be 9 Inches, you will find it against 16 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, than a little above 13 Inches will be the Length of a Foot, &c.



## § II. Of SQUAR'D TIMBER.

**B**Y Squar'd Timber are here meant all such as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids are shew'd in § II. of Chap. 2. to which I refer you.

*Example*

238 *The Mensuration of* Part II.

*Example 1.* If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) square and 18 Feet long, How many solid Feet are contain'd therein?

|                                                                                                                                                                                                   |                                                                                                                                                                                                                      |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  15 \\  15 \\  \hline  75 \\  15 \\  \hline  225 \\  18 \\  \hline  1800 \\  225 \\  \hline  144)4050(28.125 \\  \hline  1170 \\  \hline  180 \\  360 \\  720  \end{array}  $ | $  \begin{array}{r}  \text{F. I.} \\  1 \ 3 \\  1 \ 3 \\  \hline  1 \ 3 \\  \phantom{1} 3 \ 9 \\  \hline  1 \ 6 \ 9 \\  \phantom{1} 6 \\  \hline  9 \ 4 \ 6 \\  \phantom{9} 3 \\  \hline  28 \ 1 \ 6  \end{array}  $ |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Answer, 28 Feet and half a Quarter.

Here, instead of multiplying by 18, where I wrought by Feet and Inches) I multiply'd by 6, and then by 3, because 3 times 6 is 18.

*Example 2.* If a Piece of squar'd Timber be 2 Feet 9 Inches deep, and 1 Foot 7 Inches broad, and 16 Feet 9 Inches long, How many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together, and the Product will be the Content.

|       |           |
|-------|-----------|
| 33    | F. I.     |
| 19    | 2 9       |
| <hr/> | 1 7       |
| 297   | <hr/>     |
| 33    | 2 9       |
| <hr/> | 1 7 3     |
| 627   | <hr/>     |
| 16.75 | 4 4 3     |
| <hr/> | 16 8 0    |
| 3135  | <hr/>     |
| 4389  | 69 9 0    |
| 3762  | 3 3 2 3   |
| 627   | <hr/>     |
| <hr/> | 72 11 2 3 |

144)10502.25(72.93

422  
1342  
465  

---

33

Answer 72 Feet 11 Inches ; or, 72 Feet 93 Parts.

*By Scale and Compasses.*

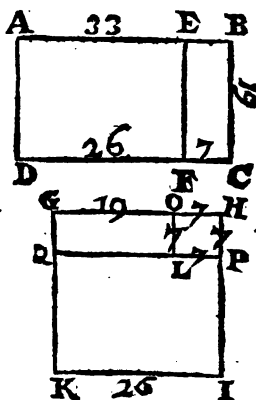
For the first Example, extend the Compasses from 12 to 15 Inches, (the Side of the Square) that Extent will reach from 18 Feet (the Length being twice turn'd over) to 28 Feet, and something more.

For the second Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts ; and the Compas Point will rest upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compasses from 12 to 25, (the Proportional found) that Extent will reach (being twice turn'd over) from 16.75 Feet, the Length, to 72.93 Feet, the Content. A

\* A common Error is committed, for want of Art, in measuring these last Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but small, when the Depth and Breadth are pretty near equal; yet if the Difference be great, the Error is very considerable; for the Piece of Timber thus measur'd, will be more than the Truth, by a Piece whose Length is equal to the Length of the Piece of Timber to be measur'd, and the Square equal to half the Difference of the Breadth and Depth, as I shall here demonstrate.

I say, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEF D; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Square is greater than the Parallelogram by the little Square OHPL; which was to be prov'd.



Otherwise, you may prove it by Numbers, thus; the Sum of 33 and 19 is 52; the half thereof is 26; the Square of 26 is 676: And the Product of the Depth and Breadth is 627; the Difference of these two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the Half thereof is 7, whose Square is 49; which was to be prov'd.

Now, if this 49 be multiply'd by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and those Feet added to the true Content, the

Sum

**Chap. 4. Squar'd Timber. 241**

Sum will be equal to the Content found by the false Way mentioned.

See the Work of both.

|                    |                                    |
|--------------------|------------------------------------|
| 33 Depth.          | 16.75 the Length.                  |
| 19 Breadth.        | 49 the Squ. of $\frac{1}{2}$ Diff. |
| <hr/>              | <hr/>                              |
| 52 Sum.            | 15075                              |
| <hr/>              | 6700                               |
| 26 half            | <hr/>                              |
| 26                 | 144)820.75(5.69                    |
| <hr/>              | <hr/>                              |
| 156                | 1007                               |
| 52                 | 1435                               |
| <hr/>              | <hr/>                              |
| 676                | 139                                |
| 16.75              |                                    |
| <hr/>              |                                    |
| 3380               |                                    |
| 4732               |                                    |
| 4056               |                                    |
| 676                |                                    |
| <hr/>              |                                    |
| 144)11323.00(78.63 |                                    |
| <hr/>              |                                    |
| 1243               |                                    |
| 910                |                                    |
| 460                |                                    |
| <hr/>              |                                    |
| 28                 |                                    |

Feet.

To 72.93 the true Content.  
Add 5.69 the Part superfluous.

Rem. 78.62 equal to the Content by the false Way.

242

*The Mensuration of**Part II.**By Feet and Inches.*

F. I.

|   |   |
|---|---|
| 0 | 7 |
| 0 | 7 |

|    |   |   |
|----|---|---|
| 0  | 4 | 1 |
| 16 | 9 |   |

|   |   |   |
|---|---|---|
| 5 | 5 | 4 |
| 3 | 0 | 9 |

|    |    |   |   |
|----|----|---|---|
| 5  | 8  | 4 | 9 |
| 72 | 11 | 2 | 3 |

Part superfluous.

true Content add.

78 7 7 0 equal to the Content by the false Way.

F. I.

|   |   |
|---|---|
| 2 | 2 |
| 2 | 2 |

|   |     |
|---|-----|
| 4 | 4   |
|   | 4 4 |

|    |   |   |
|----|---|---|
| 4  | 8 | 4 |
| 16 | 9 |   |

|    |   |   |
|----|---|---|
| 75 | 1 | 4 |
| 3  | 6 | 3 |

False C. 78 7 7

*To find how much in Length makes a Foot of any squar'd Timber.*

Always divide 1728 (the solid Inches in a Foot) by the Area of the Base; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickne's from End to End, whether it be square, triangular, multangular, or round.

*Example 1.* If a Piece of Timber be 18 Inches square, How much in Length will make a Foot solid?

|      |
|------|
| 18   |
| 18   |
| 144  |
| 18   |
| 324  |
| 1728 |
| 1620 |
| 108  |

Answer, 5 Inches and  $\frac{1}{2}$ .

By

*By Scale and Compasses.*

Extend the Compasses from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Base; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches, and  $\frac{1}{2}$  of an Inch.

Or thus: Extend the Compasses from 18 to 41.569, that Extent, turn'd twice over from 1, will at last fall upon  $5\frac{1}{2}$ , as before.

*Note,* That 41.569 is the Square Root of 1728.

*Example 2.* If a Piece of Timber be 22 Inches deep, and 15 Inches broad, How much in Length will make a Foot?

$$\begin{array}{r}
 22 \\
 15 \\
 \hline
 110 \\
 22 \\
 \hline
 330 \overline{)1728(5.23} \\
 \underline{780} \\
 1200 \\
 \hline
 210
 \end{array}$$

Answer, 5 Inches and .23 Parts.

*By Scale and Compasses.*

Extend the Compasses from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

\* There is a Line for this Purpose upon most ordinary Rules, with a little Table at the End, of all such Numbers as exceed the Length of the Rule, such as this annex'd.

|     |    |    |   |   |   |    |   |   |                 |
|-----|----|----|---|---|---|----|---|---|-----------------|
| 0   | 0  | 0  | 0 | 9 | 0 | 11 | 3 | 9 | Inches.         |
| 144 | 36 | 16 | 9 | 5 | 4 | 2  | 2 | 1 | Feet.           |
| 1   | 2  | 3  | 4 | 5 | 6 | 7  | 8 | 9 | Side of the Sq. |

Here you see, if the Side of the Square be 1, the Length must be 144 Feet; if two Inches be the Side of the Square, it must be 36 Feet in Length, to make a solid Foot, &c.

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it against 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compasses, and run the Compasses along the Piece, from End to End, you will find how many Feet are contain'd in that Piece; or you may cut off any Number of solid Feet that shall be desir'd; but if the Sides of the Piece be unequal, find a mean proportionable Number, as is before taught, by dividing the Distance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers, into two equal Parts, and you will find the middle Point at 15; so is 15 Inches the geometrical mean Proportional sought; then if you look for 15 upon the Line above-mention'd, you will find 7 Inches, and a little above half, to be the Length of a Foot.



## § III. Of unequal squar'd Timber. .

**B**Y unequal squar'd Timber, I mean all such as have unequal Bases; that is, such as is thicker at one End than at the other; and such are most Timber-trees when they are hewn, and brought to their Squares.

The usual Way to measure such Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is something near the Truth; but when there is a great Disproportion between the Ends of the Piece, the Error is considerable. All such Solids being the Frustums of Pyramids, the true Way of measuring them must be by Sect. VII. Chap. 2. I shall give an Example or two, which I will work both by the true and false Ways, whereby you will see the Difference.

*Example 1.* If a Piece of Timber be 25 Inches square at the greater End, and 9 Inches square at the lesser End, and 20 Feet long, How many Feet of Timber are in that Tree?

$$\begin{array}{r}
 25 \\
 \underline{9} \\
 \text{Sum } 34 \\
 \text{Half } 17 \text{ the Side of the Square in the Middle.} \\
 \underline{17} \\
 119 \\
 \underline{17} \\
 289 \\
 \underline{20} \\
 144 \overline{)5780(40.13} \\
 \underline{0200} \\
 \underline{560} \\
 128
 \end{array}$$

Answer, 40.13 Feet, by the false Way.

By Rule II. Sect. VII. Chap. 2.

$$\begin{array}{r}
 25 \\
 \underline{9} \\
 225 \\
 \\
 16 \text{ Difference of the Sides.} \\
 \underline{16} \\
 96 \\
 \underline{16} \\
 3)256 \text{ the Square.} \\
 \underline{\phantom{00}85} \phantom{333} \\
 225 \\
 \underline{\phantom{00}85} \phantom{333} \\
 310.333 \\
 \phantom{00}20 \\
 \hline
 144)6206.660(43.101 \\
 \underline{\phantom{00}446} \phantom{000} \\
 \phantom{00}146 \phantom{000} \\
 \phantom{00}260 \\
 \hline
 \phantom{00}116
 \end{array}$$

Answer, 43.101 Feet, by the true Way; so that there is near 3 Feet Difference.

*By Scale and CompASSES.*

Extend from 1 to 9, that Extent will reach from 25 (the same Way) to 225, the Rectangle of the Sides of the two Bases; then the Difference between the said Sides is 16; extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: then extend from 144 to 310.333, that Extent

**Chap. 4.**      *Squar'd Timber.*      **247.**

Extent will reach from 20 (the Length) to 43.1 Foot the Content the true Way.

Extend the Compasses from 12 to 17, (the Side of the middle Square) that Extent will reach from 20, (the Length being twice turn'd over) to 40.1 Foot, the Content by the false Way.

*Example 2.* If a Piece of Timber be 32 Inches broad, and 20 Inches deep, at the greater End, and 10 Inches broad, and 6 deep at the lesser End, and 18 Foot long; How many Feet of Timber are in that Piece?

**Rule I. Sect. VII. Chap. 2.**

|                                                       |                                                       |
|-------------------------------------------------------|-------------------------------------------------------|
| 32                                                    | 6                                                     |
| 20                                                    | 10                                                    |
| <hr style="width: 50px; border: 0.5px solid black;"/> | <hr style="width: 50px; border: 0.5px solid black;"/> |
| 640                                                   | 60                                                    |
| 60                                                    |                                                       |
| <hr style="width: 50px; border: 0.5px solid black;"/> |                                                       |
| 38400                                                 | (195.959 mean Proportional                            |
| 1                                                     | 640 the greater Base.                                 |
| <hr style="width: 50px; border: 0.5px solid black;"/> | 60 the lesser Base.                                   |
| 29)284                                                |                                                       |
| 385)2300                                              | 895.959 the Sum.                                      |
| 3909)37500                                            | 6 $\frac{1}{2}$ Height.                               |
| 39185)231900                                          |                                                       |
| 391909)3597500                                        | 5375.754                                              |
| 144)5375.754(37.33                                    |                                                       |
| <hr style="width: 50px; border: 0.5px solid black;"/> |                                                       |
| 1055                                                  |                                                       |
| 477                                                   |                                                       |
| 455                                                   |                                                       |
| <hr style="width: 50px; border: 0.5px solid black;"/> |                                                       |
| 23                                                    |                                                       |

And

248.

*The Mensuration of* Part II.

$$\text{Add } \left\{ \begin{array}{r} 32 \\ 10 \end{array} \right\} \text{Add}$$

$$\text{Sum } \begin{array}{r} 42 \\ \hline \end{array} \quad \begin{array}{r} 20 \\ 6 \end{array} \text{Sum.}$$

$$\text{Half } \begin{array}{r} 21 \\ \hline \end{array} \quad \begin{array}{r} 13 \\ \hline \end{array} \text{Half.}$$

$$13$$

$$63$$

$$21$$

$$\begin{array}{r} 273 \\ \hline \end{array} \text{Area in the Middle.}$$

$$18 \text{ Length.}$$

$$144)4914(34.12$$

$$1284$$

$$273$$

$$594$$

$$180$$

$$360$$

$$72$$

Feet.

$$\text{Answer } \left\{ \begin{array}{l} \text{Content the true Way } 37.33 \\ \text{Content the false Way } 34.12 \end{array} \right.$$

*By Scale and Compasses.*

Extend the Compasses from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Base.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the lesser Base: Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, so you will find the middle Point at 195.959, the Root sought; which

is

is a mean Proportional between the greater and lesser Areas; then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiply'd by 6 (a third Part of the Length), by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the false Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Base: Then extend from 144 to 273, that Extent will reach from 18, the Length, to 34.12, the Content the false Way.



#### § IV. *Of Round Timber, whose Bases are equal.*

**T**HE usual Way to measure round Timber-trees is to girt them about the Middle with a String, and take the fourth Part of that Girth for the Side of a Square, by which they measure the Piece of Timber as if it was square.

But that this is an Error, I shall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which squar'd makes .0625; this they take for a mean Area, instead of .07958: Therefore the true Content always bears such Proportion to the Content found by the aforesaid customary false Way, as .07958 to .0625; which is nearly as 23 to 18; so that in measuring by that customary false Way, there is above the one fifth Part lost, of what the true Content ought to be.

This

This Error, tho' it has been so often confuted, yet it is grown so customary in all Places, that there is little Hope of my prevailing with Men that are so wedded to it, to embrace the Truth: I shall therefore, in the following Examples, shew how to work both the true Way, and also the false or customary Way.

*Example 1.* If a Piece of Timber be 96 Inches in Circumference or Girth, and 18 Feet long, How many Feet of Timber are contain'd therein?

A 4th Part of 96 is 24

$$\begin{array}{r} 24 \\ \hline 96 \\ 48 \\ \hline \end{array}$$

576 Area Base,  
18

$$\begin{array}{r} 4608 \\ 576 \\ \hline 144)10368(72 \\ 1008 \\ \hline 288 \\ 288 \\ \hline \dots \end{array}$$

Or thus,

$$\begin{array}{r} \text{F. I.} \\ 2 \ 0 \\ 2 \ 0 \\ \hline 4 \ 0 \\ 18 \\ \hline 72 \ 0 \end{array}$$

Content the false Way, 72 Feet.

Then

# Chap. 4. Round Timber.

251

Then the true Way.

$$\begin{array}{r}
 96 \\
 96 \\
 \hline
 576 \\
 864 \\
 \hline
 9216 \\
 .07958 \\
 \hline
 73728 \\
 46080 \\
 82944 \\
 64512 \\
 \hline
 \end{array}$$

733.40928 the Area by Prob. 5. Sect. IX. Chap. 2.

$$\begin{array}{r}
 18 \\
 \hline
 586727424 \\
 73340928 \\
 \hline
 \end{array}$$

$$144)13201.36704(94.67$$

$$\begin{array}{r}
 241 \\
 973 \\
 1096 \\
 \hline
 \end{array}$$

The true Content 91.67 Feet.

$$88$$

## By Scale and Compasses.

Extend from 12 to 24 (the fourth Part of the Girth), that Extent turn'd twice over from 18 Feet (the Length), will at last fall upon 72 Feet, the Content the customary Way.

Extend from 42.54 to 96 (the Girth), that Extent will reach from 18 Feet, turn'd twice over, to 91.67 Feet, the true Content.

*Example*

*Example 2.* If a Piece of Timber be 86 Inches Girth, and 20 Feet long, How many Feet are contain'd therein?

The fourth Part of 86 is 21.5

|          |                  |
|----------|------------------|
| F. I. P. | 21.5             |
| 1 9 6    | 1075             |
| 1 9 6    | 215              |
| <hr/>    | 430              |
| 1 9 6    | <hr/>            |
| 1 4 1 6  | 462.25           |
| 0 10 9   | 20               |
| <hr/>    | <hr/>            |
| 3 2 6 3  | 144)9245.00(64.2 |
| 20       | <hr/>            |
| <hr/>    | 605              |
| 64 2 5 0 | 290              |
|          | <hr/>            |
|          | 20               |

The Content the false Way, 64.2 Feet.



By the true Way.

$$\begin{array}{r}
 86 \\
 86 \\
 \hline
 516 \\
 688 \\
 \hline
 7396 \\
 07958 \\
 \hline
 59168 \\
 36980 \\
 66564 \\
 51772 \\
 \hline
 588.57368 \\
 20 \\
 \hline
 244)11771.47360(81.74 \\
 \hline
 251 \\
 1074 \\
 667 \\
 \hline
 91
 \end{array}$$

The true Content, 81.74 Feet.

*By Scale and Compasses.*

Extend from 12 to 21.5, that Extent turn'd twice over from 20, will reach at last to 64.2 Feet, the Content the false Way.

Extend from 42.54 to 86, that Extent, turn'd twice over from 20, will at last fall upon 81.74 Feet, the true Content.

These Cylindrical Proportions may be very easily wrought upon the Line of Numbers.

Z

*Problem*

*Problem 1.* Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppose the Diameter 22.6 Inches.

As 22.6 is to 46.9, so is 1 to a fourth Number; and that to the Length of a Foot in Inches 4.3.

Extend the Compasses from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

*Problem 2.* Having the Diameter in Foot-measure, to find the Length of a Foot in Foot-measure.

Suppose the Diameter 1.88 Feet.

Then, as 1.88 is to 1.128, so is 1 to a fourth Number; and so is that to the Length of a Foot in Foot-measure, .358.

Extend the Compasses from 1.88 to 1.128, that Extent, turn'd twice from 1, will reach to .358 Parts of a Foot.

*Problem 3.* Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppose the Circumference 71 Inches.

Then as 71 is to 247.36, so is 1 to a fourth Number; and so is that to the Length of a Foot in Inches, 4.3.

Extend the Compasses from 71 to 247.36, that Extent, turn'd twice from 1, will reach to 4.3 Inches, the Length of a Foot.

*Problem 4.* Having the Circumference in Foot-measure, to find the Length of a Foot in Foot-measure.

Suppose the Circumference 5.92 Feet.

Then, as 5.92 is to 3.345, so is 1 to a fourth Number; and so is that to the Length of a Foot in Foot-measure, .358.

Extend

Extend the Compasses from 5.92 to 3.545, that Extent, turn'd twice over from 12 will fall upon 358 Parts of a Foot.

*Problem 5.* Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Inches, 62674.

Extend the Compasses from 1.128 to 22.6, that Extent, turn'd twice from 156, will fall upon 62674 Inches, the Content.

*Note,* That 1.128 is the Diameter when the Side of the Square is equal to 1.

*Problem 6.* Having the Diameter in Foot-measure, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 is to 1.88, so is 13 to a fourth Number; and so is that to the Content in Feet 36.27.

Extend from 1.128 to 1.88, that Extent, turn'd twice from 13, will fall upon 36.27.

*Problem 7.* Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 46.9 to 22.6, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

*Note,* That 46.9 is the Diameter of a Circle, whose Area is 1728.

*Problem 8.* Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 is to 22.6, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 13.54 to 22.6, that Extent, turn'd twice from 13, will fall upon 36.27.

*Note,* That 13.54 is the Diameter of a Circle, when the Area is 144.

*Problem 9.* Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppose the Circumference 71, and the Length 156 Inches.

Then, as 3.545 is to 71, so is 156 to a fourth Number; and so is that to 62674, the Content in Inches.

Extend the Compasses from 3.545 to 71, that Extent turn'd twice from 156, will fall upon 62674, the Content.

*Note,* That 3.545 is the Circumference, when the Side of the Square is equal to 1.

*Problem 10.* Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 is to 5.92, so is 13 to a fourth Number; and so is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turn'd twice from 13, will fall upon 36.27 Feet, the Content.

*Problem 11.* Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 is to 71, so is 156, to a fourth Number; and so is that to the Content in Feet 36.27.

Extend the Compasses from 147.36 to 71, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

*Note,* That 147.36 is the Circumference of a Circle, whose Area is 1728.

*Problem 12.* Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 is to 71, so is 13 to a fourth Number; and so is that to the Content in Feet 36.27.

Extend the Compasses from 42.54 to 71, that Extent, turn'd twice from 13, will reach to 36.27 Feet, the Content.

*Note,* That 42.54 is the Circumference of a Circle, whose Area is 144.

§ V. Of Round Timber, whose Bases are unequal.

**T**HE usual Way to measure round Timber, (as I said before) is to take a fourth Part of the Girth in the middle of the Piece, for the Side of a mean Square. But this Way I have prov'd to be erroneous in Timber that is all the Way of an equal Thickness, and it must be much more so in Timber that is tapering; and the more tapering it is, the greater is the Error: for to the Error in the last Section, there is added the Error in the third Section; therefore, to measure all such Timber according to Art and Truth, such a Piece ought to be consider'd as a Frustum of a Cone, and should be measur'd by the Rules given in Section VIII. Chapter 2. by which Rules the following Examples are wrought.

*Example 1.* If a Piece of Timber be 9 Inches Diameter at the lesser End, and 36 Inches at the other End, and 24 Feet long, How many Feet of Timber are therein?

|                                                  |                                                                                                                                                                                                               |
|--------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 36 \\ 9 \\ \hline \end{array}$ | $\begin{array}{r} 36 \\ 9 \\ \hline \end{array} \left. \vphantom{\begin{array}{r} 36 \\ 9 \\ \hline \end{array}} \right\} \text{Subtract.}$                                                                   |
| $\text{Rect. } 324.$                             | $\begin{array}{r} 27 \text{ Difference.} \\ 27 \\ \hline 189 \\ 54 \\ \hline 3 \overline{) 729} \text{ the Square.} \\ \hline 243 \text{ one Third.} \\ 324 \text{ Rectangle add.} \\ \hline 567 \end{array}$ |

$$\begin{array}{r}
 .7854 \\
 \underline{.567} \\
 54978 \\
 47124 \\
 \hline
 39270
 \end{array}$$

A Mean Area 445.3218

$$\begin{array}{r}
 24 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 17812872 \\
 8906436 \\
 \hline
 \end{array}$$

144) 10687.7232 (74.22

$$\begin{array}{r}
 607 \\
 317 \\
 \hline
 292
 \end{array}$$

4

Answer, 74.22 Feet.

Or thus, by Feet and Inches.

F. I.

$$\begin{array}{r}
 3 \ 0 \\
 0 \ 9 \\
 \hline
 \end{array}$$

2 3 Rect.

F. I.

$$\begin{array}{r}
 2 \ 3 \text{ Difference.} \\
 2 \ 3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 4 \ 6 \\
 0 \ 6 \ 9 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 5 \ 0 \ 9 \text{ the Square.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1 \ 8 \ 3 \text{ one Third.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 2 \ 3 \ 0 \text{ Rectangle added.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3 \ 11 \ 3 \text{ a mean Square.} \\
 \hline
 \end{array}$$

Then;

P. I. P.

Then, as 14 is to 11, so is 3, 11 3 to the Area.

$$\begin{array}{r}
 \hline
 7 \overline{) 43 \ 3 \ 9} \\
 \hline
 2 \overline{) 6 \ 2 \ 3} \\
 \hline
 3 \ 1 \ 1 \ 6 \\
 \hline
 18 \ 6 \ 9 \ 0 \\
 \hline
 4 \\
 \hline
 74 \ 3 \ 0 \ 0
 \end{array}$$

Here, instead of dividing by 14, I divide by 7 and by 2, because twice 7 is 14.

And instead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, because 6 times 4 is 24.

By Scale and Compasses this is too troublesome.

*Example 2:* If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long, How many Feet of Timber are contain'd in that Piece?



Chap. 4.

Round Timber.

261

136  
32  

---

272  
408  

---

4352

136  
32  

---

104 Difference.  
304  

---

416  
1040

3)10816 the Square.

3605.333 one Third.  
4352 Rectangle add.

7957.333 a mean Circumf. squar'd.  
07958.

63658664  
39786665  
71615997  
55701331.

633.24456014 the mean Area.  
21

63324456014  
126648912028

13298.13576294

144)13298.13(92:34

338  
501  
693

117  
Answer, 92:34 Feet.

B<sub>2</sub>

By Feet and Inches, thus ;

| F.    | I.  | F.    | I.              |
|-------|-----|-------|-----------------|
| 11    | 4   | 8     | 8 Difference.   |
| 2     | 8   | 8     | 8               |
| <hr/> |     | <hr/> |                 |
| 22    | 8   | 69    | 4               |
| 7     | 6 8 | 5     | 9 4             |
| <hr/> |     | <hr/> |                 |
| 30    | 2 8 | 3)75  | 1 4 the Square. |

|       |   |   |   |
|-------|---|---|---|
| 25    | 0 | 5 | 4 |
| 30    | 2 | 8 | 0 |
| <hr/> |   |   |   |

55 3 1 4 the Sq. of the Circum.

F. I. P. S.  
88 : 7 :: 55 3 1 4 : the mean Area.  
7

11)386 9 9 4

8)35 1 11 9

4 4 8 11 the mean Area.  
7

30 9 2 5  
2

*Facit* 92 3 7 3

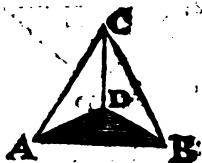
§ VI. *Of the Five Regular Bodies.*

**T**HESE Bodies may all be measur'd by the 4th Section of Chap. 2. except it be the Cube, or Hexaedron, which is already measur'd in Section I. of that Chapter.

*Of the TETRAEDRON.*

A Tetraedron is a Solid, contain'd under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whose Side is 12 Inches, the perpendicular Height 9.798 Inches.



By Sect. V. Chap. 1. the Area of the Triangle will be found 62.352; a third Part of it is 20.784, which multiply'd, by the perpendicular Height, the Product is 203.641632 solid Inches, the Content.

10.392 the Perpendicular of the Triangle.  
6 half the Side.

---

62.352 Area of the Triangle.

---

20.784 one third Part.  
9.798 the perpendicular Height.

---

166272  
187056  
145488  
187056

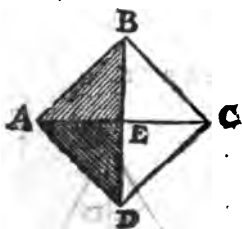
---

203,641632

The

The superficial Content is four times the Area of the Triangle, *viz.* 249.408 Inches, because there are 4 Triangles.

## 2. Of the OCTAEDRON.



The Octaedron is a Body contain'd under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whose Side is 12 Inches; the Content solid and superficial is requir'd.

An Octaedron is compos'd of two quadrangular Pyramids join'd together by their Bases; therefore, if the Area of the Base be multiplied into a third Part of the Length of both Pyramids, the Product will be the solid Content.

5.6568 a third Part of the Length.  
144 Area of the square Base.

---

226272  
226272  
56568

---

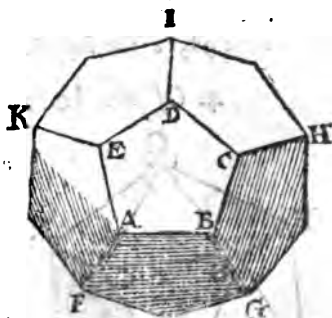
814.5792 the Solidity.

The superficial Content will be just double to that of the Tetraedron, *viz.* 498.816, because the Side of this is suppos'd to be equal to the Side of that; and because the Octaedron is contain'd under eight Triangles, and the Tetraedron but under four.

3. *Of the DODECAEDRON.*

The Dodecaedron is a solid Body contained under twelve pentangular Plains.

Let ABCDEFGHIK be a Dodecaedron, each Side thereof being 12 Inches; the Content solid and superficial is required.



The Solidity of the Dodecaedron is compos'd of 12 pentangled Pyramids, whose Vertexes all meet in the Centre.

Therefore, if we find the Solidity of one of those Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292

30 half Sum of the Sides.

247.748760 Area of the Pentagon.

60454.4 A third Part of 13.36219 inverted.

99099504

9909950

1238744

99099

1486

1103.48783 Content of one Pyramid.

12

13241.85396 The Solidity of the Dodecaedron.

A a

15

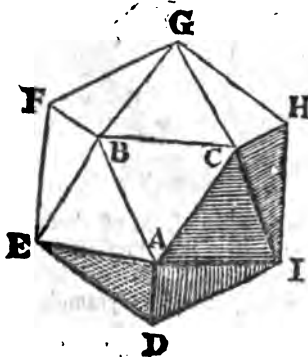
If the Area of the Pentagon be multiply'd by 12.  
the Product will be the superficial Content.

247.74876

12

2972.98512 the superficial Content.

#### 4. Of the ICOSAEDRON.



The Icofaedron is a solid Body, contain'd under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icofaedron, each Side thereof being 12 Inches; the Content solid and superficial is required.

The Icofaedron is compos'd of twenty triangular Pyramids, with their Vertexes all joining in the Centre.

Therefore, if the solid Content of one Pyramid be multiply'd by 20, the Product is the whole solid Content of the Icofaedron.

**Chap. 4. *the Five Regular Bodies.* 267**

10.39224 the Perpendicular of the Triangle.  
6 half the Side.

---

62.35344  
20

---

1247.06880

3.0230456 third Part of the Altitude of the Pyramid  
44353.26

---

181382736  
6046091  
906914  
151152  
9069  
1209  
121

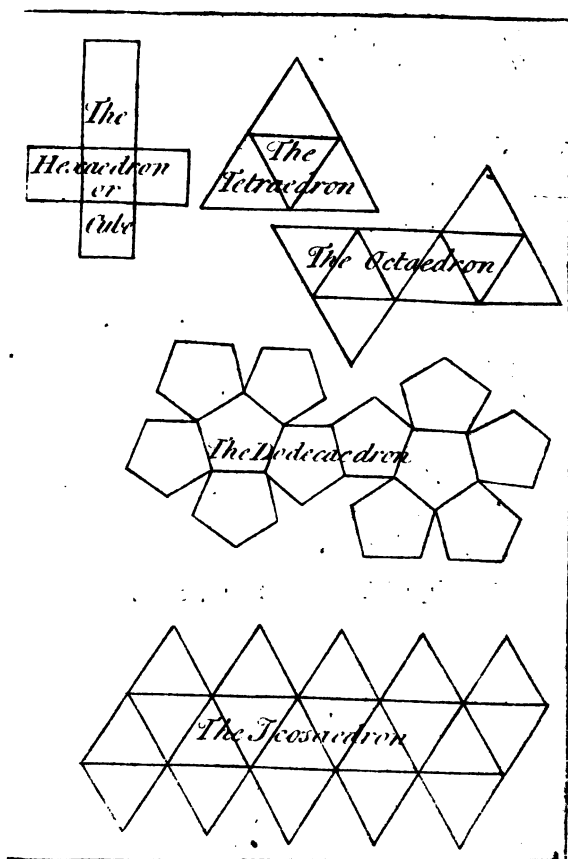
---

188.497292  
20

---

3769.945840 the Solidity.

The superficial Content 1247.0688.



By these Figures you may cut these Bodies in fine  
Pastboard, cutting all the Lines half through, and so  
turn them up and glue them.



*A TABLE shewing the Solidity and Superficial Content of any of the regular Bodies, the Side being 1, or Unity.*

| The Names of the Bodies. | The Solidity. | Superficies. |
|--------------------------|---------------|--------------|
| Tetraedron               | 0.1178511     | 1.732051     |
| Octaedron                | 0.4714045     | 3.464102     |
| Hexaedron                | 1.0000000     | 6.000000     |
| Icosaedron               | 2.181695      | 8.660254     |
| Dodecaedron              | 7.663119      | 20.645729    |

By this Table, the Content, either superficial or solid, of any of these Bodies, may very readily be found ; for all like superficial Figures are in Proportion one to another, as are the Squares of their like Sides : Therefore it will be, As the Square of 1 (which is 1) is to the superficial Content in the Table ; so is the Square of the Side of the like Body, to the superficial Content of the same Body. Therefore, if the Number in the Table be multiply'd by the Square of the Side given, the Product is the superficial Content requir'd.

Again, all like Solids are in such Proportion to each other, as are the Cubes of their like Sides. Therefore it will be as 1 (which is the Cube of 1) is to the solid Content in the Table, so is the Cube of the Side given, to the solid Content requir'd. Therefore, if the Number in the Table be multiply'd by the Cube of the given Side, the Product will be the solid Content of the same Body.

## 270 *The Mensuration of* Part II.

*Example 1.* If the Side of a Dodecaedron be 12 Inches (as before), what is the Content solid and superficial?

7.663119 the tabular Number.  
1728 the Cube of the Side.

---

61304952  
15326238  
53641833  
7663119

---

13241.869632 the solid Content, nearly the same as  
(before.

20.645729 the tabular Number.  
144 the Square of the Side.

---

82582916  
82582916  
20645729

---

2972.984976 the superficial Content.

### *By Scale and Compasses.*

Extend from 1 to 12 (the Side), that Extent being turn'd three times over from 7.63119, will at last fall upon 13241.86, &c. the solid Content.

And if you apply the same Extent twice from 20.645729, it will at last fall upon 2972.98, &c. the superficial Content.

*Example*

## Chap. 4. *the Five Regular Bodies.* 271

*Example 2.* If the Side of an Octaedron be 20 Inches, what is the Content solid and superficial?

.4714045 the tabular Number.  
8000 the Cube of the Side.

---

{3771.2360000 the solid Content.

---

3.464102 the tabular Number.  
400 the Square of the Side.

---

1385.640800 the superficial Content.

### *By Scale and Compasses.*

Extend from 1 to 20, that Extent, turn'd three times over from .4714045, will at last fall upon 3771.236, the solid Content. The same Extent turn'd twice over from 3.464, &c. will at last fall upon 1385.64, the superficial Content.



## § VII. *How to 'measure' any irregular Solid.*

**I**F you have any Piece of Wood or Stone that is craggy and uneven, and you desire to find the Solidity, put the Solid into any regular Vessel, as a Tub, a Cistern, or the like, and pour in as much Water as will just cover it; then take out the Solid, and measure how much the Fall of the Water is, and to find the Solidity of that Part of the Vessel.

*Example*

*Example.* Suppose a Piece of Wood or Stone to be measur'd, and suppose a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and cover'd with Water; then, when the Solid is taken out, suppose the Fall of the Water 14 Inches; square 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Base; which multiply'd by 14, the Depth or Fall of the Water, and the Product is 11259.49, &c. which divided by 1728, the Quotient is 6.51 Feet; and so much is the solid Content requir'd.





# CHAP. V.

## *Practical Questions in MEASURING.*

*Question 1.* IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contained therein ?

| F.    | I.   |
|-------|------|
| 47    | 9    |
| 18    | 6    |
| <hr/> |      |
| 376   | 0    |
| 47    |      |
| 23    | 10 6 |
| 9     | 0 0  |
| 4     | 6 0  |
| <hr/> |      |
| 883   | 4 6  |

| F.    | I.      |
|-------|---------|
| 47    | 75      |
| 18    | 5       |
| <hr/> |         |
| 23875 |         |
| 38200 |         |
| 4775  |         |
| <hr/> |         |
| 9     | 883.375 |
| <hr/> |         |
|       | 98.1    |

Answer, 98 Yards 1 Foot.

*Quest.*

*Quest. 2.* There is a Room, whose Length is 21.5 Feet, and the Breadth 17.5 Feet, is to be pav'd with Stones, each 18 Inches square: I demand how many such Stones will pave it?

|                 |                         |
|-----------------|-------------------------|
| 21.5            | 1.5                     |
| 17.5            | 1.5                     |
| <hr/>           | <hr/>                   |
| 1075            | 75                      |
| 1505            | 15                      |
| 215             | <hr/>                   |
| <hr/>           |                         |
| 2.25)376.25(167 | 2.25 Area of one Stone. |
| <hr/>           |                         |
| 1512            |                         |
| 1625            |                         |
| <hr/>           |                         |

50 Answer, 167 Stones.

*Quest. 3.* There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad) to be hung with Tapestry that is Ell-broad: I desire to know how many Yards will hang the said Room?

From the Content of the Room, subtract the Content of the Windows, and divide the Remainder by the square Feet in a Yard of Tapestry.

Chap. 5. *Practical Questions.* 275

|      |        |          |      |
|------|--------|----------|------|
| 3.75 | 109.75 | Length.  | 5.75 |
| 3    | 9.25   | Breadth. | 6.5  |

|       |       |      |
|-------|-------|------|
| 11.25 | 54875 | 2875 |
|       | 21950 | 3450 |
|       | 98775 |      |

1015.1875 Content of the Room. 37.375

74.75 Content of the Windows sub. 2

11.25)940.4375(83.59 74.750

4043

6687

10625

500

Answer, 83.59 Yards.

*Quest. 4.* If the Axis of a Globe be 27.5 Inches, I demand the Content, solid and superficial.

3.1416

27.5

157080

219912

62

86.39,400 the Circumference.

86.394

86.394 the Circumference.  
27.5 the Diameter.

---

431970  
604758  
172788

---

672375.8350 the superficial Content.

---

395.9725 a sixth Part.  
27.5

---

19798625  
27718075  
7919450

---

10889.24375

Answer,  $\left\{ \begin{array}{l} 6.3 \text{ Feet solid} \\ 16.49 \text{ Feet superficial.} \end{array} \right.$

*Quest. 5.* There is the Frustrum of a Globe, the Diameter of whose Base is 24 Inches, and the Altitude thereof is 10 Inches; what is the Content solid and superficial?

Find the Superficies as is directed in Page 188, and 24 the Solidity by the first Theorem in Page 190.

|       |       |           |     |
|-------|-------|-----------|-----|
| 24    | .7854 | .7854     | 20  |
| 96    | 576   | 400       | 20  |
| 48    |       |           |     |
| <hr/> | 47124 | 314.16000 | 400 |
| 576   | 54978 |           |     |
|       | 39270 |           |     |

452.3904 } add.  
314.16

766.5504 the Curve Superficies.  
452.3904 the Base add

---

1218.9408 } the whole superficial  
Content in Inches.



$$12 \times 12 = 144$$

3

432

100 the Square of the Alt. add.

532

the Sum.

10 multiply by the Alt.

5320

5236 multiply.

31920

15960

10640

26600

2785.5520 the Solidity in Inches.

*Quest.* 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; how many Tons of Timber are contain'd in that Tree?

F. I.

4)18 6 the Girth.

4 7 6 a 4th Part

4 7 6

18 6 0

2 8 4 6

2 3 9

21 4 8 3

6

128 4 1 6

4

4|051|3 4 6 0

12 33

Answer, 12 Tons 33 Feet 4 Inches 6 Parts.

B b

*Note,*

Here I multiply by 6 and by 4, because 6 times 4 is 24.

*Note,* That 40 Feet of Timber is a Ton, and 50 Feet a Load.

*Note also,* That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-wood, that is, 128 cubical Feet.

*Quest. 7.* There is a Cellar to be dug by the Floor, whose Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar.

F. I.

33 7 the Length.

18 9 the Breadth.

---

264

33

16 9 6

8 4 9

9 0 0

1 6 0

---

629 8 3

5 9 0 the Depth.

---

3148 5 3

314 10 1

157 5 0

---

324)3620 8 4(11

380

---

56

Answer, 11 Floors 56 Feet.

*Note,* That 18 Feet square, and a Foot deep, is a Floor of Earth, that is, 324 solid Feet.

*Quest.*

**Chap. 5. Practical Questions. 279**

*Quest. 8.* There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; How many Squares of Tiling are contain'd therein?

|     |    |     |
|-----|----|-----|
| F.  | I. |     |
| 48  | 9  |     |
| 35  | 6  |     |
|     |    |     |
| 240 |    |     |
| 144 |    |     |
| 24  | 4  | 6   |
| 17  | 6  | 0   |
| 8   | 9  | 0   |
|     |    |     |
| 17  | 30 | 7 6 |

Answer, 17 Squares 30 Feet.

*Quest. 9.* There is a Cone, whose Diameter at the Base is 42 Inches, and the perpendicular Height 94 Inches, and it is requir'd to cut off two solid Feet from the top End thereof; I demand what Length upon the Perpendicular must be cut off?

|               |       |                  |
|---------------|-------|------------------|
| 42            | 1728  | 94               |
| 42            | 2     | 94               |
| <hr/>         | <hr/> | <hr/>            |
| 84            | 3456  | 376              |
| 168           |       | 846              |
| <hr/>         |       | <hr/>            |
| 1764          |       | 8836 Square.     |
| .7854         |       | 94               |
| <hr/>         |       | <hr/>            |
| 7056          |       |                  |
| 8820          |       | 35344            |
| 14112         |       | 79524            |
| 12348         |       | <hr/>            |
| <hr/>         |       | 830584 the Cube. |
| 1385.4456     |       |                  |
| 94            |       |                  |
| <hr/>         |       |                  |
| 55417824      |       |                  |
| 124690104     |       |                  |
| <hr/>         |       |                  |
| 3)130231.8864 |       |                  |

43410.6288

All the solid Bodies are in triplicate Reason of their homologous Sides by *Euc.* 12, 12; 12, 18; and 11, 33: therefore it will be,

Solidity of the Cone.    Cub. Alt.    Solidity of 2 Feet.

43410.6288 : 830584 :: 3456 :

3456

4983504

4152920

3322336

2491752

434106288)2870498304(66124 the Cube of the Length.

265860576

5396803

1055740

187528

66124

$$\begin{array}{r} 66124(40.43 \\ 64 \end{array}$$

2124000 Resolvend.

$$\begin{array}{r} 12 \\ 48 \end{array}$$

492 Divisor.

$$\begin{array}{r} 120 \\ 4800 \end{array}$$

48120 Divisor.

$$\begin{array}{r} 64 \\ 1920 \\ 19200 \end{array}$$

1939264 Subtrahend.

184736000 Resolvend:

$$\begin{array}{r} 1212 \\ 489648 \end{array}$$

4897692 Divisor.

$$\begin{array}{r} 27 \\ 10908 \\ 1468944 \end{array}$$

147003507 Subtrahend.

$$37732493$$

*Answer.* The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greatest End, then from 43410.6288 subtract 3456, and the Remainder is 39954.6288. Then say,

$$43410.6288 : 830584 :: 39954.6288$$

830584

$$\begin{array}{r} 1598185152 \\ 1598185152 \\ 3196370304 \\ 1997731440 \\ 11986388640 \\ 3196370304 \end{array}$$

$$13410.6288)33185675407.2192(764459(91.4$$

..... 729

$$279823524$$

$$19359751 \quad 35459$$

$$1995500$$

$$259075 \quad 27$$

$$42022 \quad 243$$

$$2952 \quad 2457$$

$$271$$

$$243$$

$$24571$$

$$10888000$$

$$273$$

$$24843$$

$$248703$$

*Answer.* It must be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

*Quest.*

*Quest. 10.* If a square Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Base be 21 Inches, and the Side of the Square of the lesser Base be 3 Inches; How far must I measure from the greater End, to cut off 5 solid Feet?

First, find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18: Then,

Diff. Length. great. Length

As 18 : 12 :: 21 : 14

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches.

The solid Content of the whole Pyramid is 24896 Inches, and the solid Content of 5 Feet is 8640; which subtracted from 24896, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

24896 : 4741632 :: 16056 :

3082752, whose Cube Root is 145.54; subtract this Root from 168 (the Length), and there remain 22.46 Inches, which is the Length of 5 solid Feet at the great End.

*Quest. 11.* Three Men bought a Grinding-stone of 40 Inches Diameter, which cost 20 Shillings; of which Sum the first Man paid 9 Shillings, the second 6 Shillings, and the third 5 Shillings: I demand how much of the Stone each Man must grind down, proportionable to the Money he paid?

All Circles are in duplicate Reason of their Diameter, by *Euc. 12, 2.*

Square the Semidiameter, which makes 400. Then

s. Square. s.

20 : 400 :: 9 : 180

This 180 is the Square of the Semidiameter of the Circle belonging to the first Man.

$$\text{And, } 20 : 400 :: 6 : 120$$

This 120 is the Square of the Semidiameter of the Circle belonging to the second.

$$\text{And, } 20 : 400 :: 5 : 100$$

This 100 is the Square of the Semidiameter of the Circle belonging to the third.

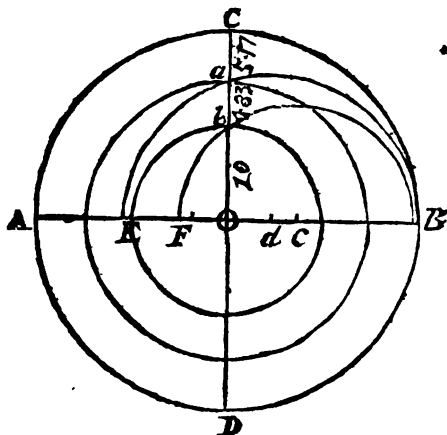
Then, from 400 (the Square of the Semidiameter of the Stone) subtract 180; and there remains 220, whose square Root is 14.83 Inches; which subtracted from 20 Inches (the Semidiameter), there remain 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which must be ground down by the first.

Then, from 220 subtract 120, and there remains 100, whose square Root is 10; subtract that from 14.83, and there remain 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the second Man. The third must grind down the Remainder, which is 10 Inches, the square Root of 100.

This Question may very easily and speedily be perform'd Geometrically, as in the annexed Scheme.

First,





First, upon the Centre  $\odot$  strike the Circle ABCD, and cross it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A  $\odot$ , (which suppose 20) in Proportion to 9s. 6s. and 5s. (the several Sums paid by the three Men) by the Points E and F; so shall AE be 9, EF 6, and F  $\odot$  5: Then divide EB into 2 equal Parts in d, and upon d, as a Centre, strike the Semicircle EaB; and divide FB into 2 equal Parts in c, and upon c, as a Centre, with the Radius cF, strike the Semicircle FbB: So have you the Semidiameter  $\odot$  C divided into three such Parts as the Stone ought to be divided; and Circles, struck thro' those Points, will shew how much each Man must grind for his Share.

*Quest. 12.* A Gard'ner he had an upright Cone,  
Out of which should be cut him a Rolling-stone,

The biggest that e'er it could make :  
The Mason he said, That there was a Rule  
For such Sort of Work, but he had a thick Skull :

**Now help him for Pity's sake.**

*Answer,* It must be cut at one third Part of the Altitude. *Quest.*

Знѣѣ.

*Quest.* 13. There is a Cistern, whose Depth is seven Tenths of the Width, and the Length is six times the Depth, and the solid Capacity is 367.5 Feet. I demand the Depth, Width, and Length, and how many Bushels of Corn it will hold ?

First, you must find three Numbers, in Proportion to the Depth, Width, and Length, thus ; suppose the Depth 7, then the Width will be 10, and the Length 42 ; which multiply'd together, the Product is 2940, which is the solid Inches in a Cistern, whose Depth is 7, Width 10, and Length 42. But the solid Inches in the Question are 635040 ( $= 367.5 \times 1728$ ) then the Cube of supposed Width is 1000. So it will be,

$2940 : 1000 :: 635040 : 216000$ , whose Cube Root is 60, which is the true Width ; 7 Tenths thereof is 42, the Depth ; and 6 times 42 is 252 Inches, the Length ; which three Numbers being multiply'd together, the Product will be 635040. If these solid Inches be divided by 2150.42, and the Quotient is 295  $\frac{44}{100}$  Bushels, or 36 Quarters 7 Bushels 1 Peck 4 Pints. And so much will the Cistern hold.

*Quest.* 14. Suppose, Sir, a Bushel be exactly round, Whose Depth being measur'd, 8 Inches is found, If the Breadth 18 Inches and half you discover, This Bushel is legal all England over.  
But a Workman would make one of another Frame ; Sev'n Inch and a half must be the Depth of the same : Now, Sir, of what Length must the Diameter be, That it may with the former in Measure agree ?

# Chap. 5. *Practical Questions.*

287

18.5  
18.5

925  
1480  
185

342.25 the Square.  
.7854

136900  
171125  
273800  
239575

268.898150

8

2150.425200 the solid Inches in a Bushel.

7.5)2150.4252(286.72336

650  
504  
542  
175  
252  
270  
450  
0

.7854

$$\begin{array}{r} .7854)286.72336 \\ \hline \end{array}$$

$$\begin{array}{r} 51103 \\ \hline \end{array}$$

$$\begin{array}{r} 39793 \\ \hline \end{array}$$

$$\begin{array}{r} 52360 \\ \hline \end{array}$$

$$\begin{array}{r} 5236 \\ \hline \end{array}$$

$$\begin{array}{r} .365.0666(19.107 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \hline \end{array}$$

$$\begin{array}{r} 29)265 \\ \hline \end{array}$$

$$\begin{array}{r} 261 \\ \hline \end{array}$$

$$\begin{array}{r} 381)406 \\ \hline \end{array}$$

$$\begin{array}{r} 381 \\ \hline \end{array}$$

$$38207)256600$$

Answer, The Diameter must be 19.107 Inches, if the Depth be 7.5 Inches.

*Quest. 15.* In the midst of a Meadow well stored with Grass,

I took just an Acre to tether my As;

How long must the Cord be, that feeding all round,

He mayn't graze less nor more than his Acre of Ground?

By Problem 10. Section IX. Chap. 1. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.

66 Feet, the Breadth of an Acre.

$$\begin{array}{r} 3960 \\ \hline \end{array}$$

$$\begin{array}{r} 3960 \\ \hline \end{array}$$

43560 the square Feet in an Acre.

As 1 : 1.2132 :: 43560

43560

763920

63660

38196

50928

...

55460.5920(235.5 Diamet.

4

117.75 half.

43)154

129

465)2560

2325

4705)23559

23525

34

Answer, The Chord must be 117 Feet and 9 Inches.

*Quest.* 16. A Maltster has a Kiln, that is 16 Feet 6 Inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much square the new one must be?

$$\begin{array}{r} 16.5 \\ 16.5 \\ \hline \end{array}$$

$$\begin{array}{r} 825 \\ 990 \\ 165 \\ \hline \end{array}$$

272.25 the Area of the old one.

$$\begin{array}{r} 3 \\ \hline 816.75(28.57 \\ 4 \end{array}$$

$$\begin{array}{r} 48)416 \\ 384 \\ \hline \end{array}$$

$$\begin{array}{r} 565)3275 \\ 2825 \\ \hline \end{array}$$

$$\begin{array}{r} 5707)45000 \\ 39949 \\ \hline 5051 \end{array}$$

Answer, The Side of the new one must be 28 Feet, and near 7 Inches.

*Quest.*

# Chap. 5. *Practical Questions.*

291

*Quest.* 16. If a round Cistern be 26.3 Inches Diameter, and 52.5 Inches deep; how many Inches Diameter must a Cistern be to hold twice the Quantity, the Depth being the same? And how many Ale-Gallons will each Cistern hold?

$$\begin{array}{r}
 26.3 \\
 26.3 \\
 \hline
 789 \\
 1578 \\
 526 \\
 \hline
 691.69 \text{ the Square.} \\
 2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1383.38(37.19 \\
 9 \\
 \hline
 67)483 \\
 469 \\
 \hline
 741)1438 \\
 741 \\
 \hline
 7429)69700 \\
 66861 \\
 \hline
 2839
 \end{array}$$

The Diameter of the greater is 37.19 Inches.

691.69 the Square of the lesser Cistern's Dia.  
 .7854

---

276676  
 345845  
 553352  
 484183

---

543.253326 Area of the Base.  
 52.5

---

2716266630  
 1086506652  
 2716266630

---

28520.7996150 solid Content in Inches.

---

282)28520.799(101.137 Gallons.

---

320  
 387  
 1059  
 2139

---

165

*Note,* That 282 solid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.



# Chap. 5. *Practical Questions.* 293

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

$$\begin{array}{r}
 359.05 \overline{)1383.38} \qquad (3.853 \\
 \underline{306230} \qquad \qquad \underline{52.5} \\
 18990 \qquad \qquad 19265 \\
 \underline{1037} \qquad \qquad \underline{7706} \\
 \qquad \qquad \underline{19265}
 \end{array}$$

The Content of the greater 202.2825 Gallons.

*Quest.* 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches; How many Ale Gallons are contain'd therein?

$$\begin{array}{r}
 25 \qquad \qquad 32 \qquad \qquad 359 \\
 \underline{25} \qquad \qquad \underline{32} \qquad \qquad \underline{3} \\
 125 \qquad \qquad 64 \qquad \qquad 1077 \\
 \underline{50} \qquad \qquad \underline{96} \\
 625 \qquad \qquad 1024
 \end{array}$$

1024 Square of the Bung Diameter.

1024 the same.

625 Square of the Head Diameter.

$$\begin{array}{r}
 1077 \overline{)2673} \qquad (2.48 \\
 \underline{5190} \qquad \qquad \underline{40} \\
 8820 \qquad \qquad 99.20 \\
 \underline{204}
 \end{array}$$

Answer, 99.2 Gallons.

Otherwise you may find a mean Diameter, and work by Scale and Compasses, thus: Subtract 25 from 32, and there remains 7, which multiply'd by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compasses from 18.95 to 29.9, that Extent, turn'd twice from 40 (the Length) will fall upon 99.6 Gallons; something more than before.

*Quest. 19.* There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds.

The Cube of 20 (the Length) is 8000. Then (by *Euc. 11.33*),

217 : 8000 :: 1000 : 36870.645, whose Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then say,

$$\begin{array}{l} 20 : 33.28 :: 15 : 24.96 \\ 20 : 33.28 :: 8 : 13.312 \end{array}$$

Answer,  $\left\{ \begin{array}{l} \text{The Length} \quad 33.28 \\ \text{The Breadth} \quad 24.96 \\ \text{The Thickness} \quad 13.312 \end{array} \right\}$  Inches.

*Quest. 20.* If an iron Bullet, whose Diameter is 4 Inches, weighs 9 Pounds; What will be the Weight of another Bullet (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729. Then (by *Euc. 12.18.*),

$$\begin{array}{rcl} & \text{lb.} & \text{lb.} \\ 64 : 9 :: 729 : 102.515 \\ & \text{lb.} & \text{oz. dr.} \end{array}$$

Answer, It weighs 102 8 4 *fors.*

*Quest.*

*Quest. 21.* There is a square Pyramid of Marble; each Side of its Base is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like square Pyramid, each Side of whose Base is 30 Inches.

The Cube of 5 is 125, and the Cube of 30 is 27000. Then (by *Euc. 12. 12.*)

lb. lb.

$$125 : 12.25 :: 27000 : 2646$$

Answer, The Weight is 2646 Pounds.

*Quest. 22.* There is a Ball or Globe of Marble, whose Diameter is 6 Inches, and its Weight 11 Pounds; What will be the Diameter of another Globe of the same Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

lb. lb.

$$11 : 216 :: 500 : 9818.1818.$$

• Whose Cube Root is 21.4 Inches, the Diameter sought.

*Quest. 23.* There is a Frustum of a Pyramid, whose Bases are regular Octagons; each Side of the greater Base is 21 Inches, and each Side of the lesser Base is 9 Inches, and its Length is 15 Feet; I demand, how many solid Feet are contained therein?

4.8284 the tabular Number, Page 88.

237 the Square of a mean Side.

|                      |     |       |
|----------------------|-----|-------|
| 337988               | 21  | 11    |
| 144852               | 9   | 12    |
| 96568                | 189 | 3)144 |
| 11443308             | 48  | 48    |
| 15                   | 237 |       |
| 57216540             |     |       |
| 11443308             |     |       |
| 144)17164.9620(119.2 |     |       |
| 276                  |     |       |
| 1324                 |     |       |
| 289                  |     |       |
| 1                    |     |       |

Answer, 119.2 solid Feet.

*Ques.* 24. There is a Frustum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustum.

First, Find the Length of the whole Cone, thus:

|           |
|-----------|
| From 36   |
| Subtr. 20 |
| 16        |

$$16 : 215 :: 36 : 483.75$$

So the Length of the whole Cone is  $483\frac{3}{4}$  Inches.

Then

Then find the Content of the whole Cone.

|                                                       |                                                       |
|-------------------------------------------------------|-------------------------------------------------------|
| 36                                                    | 1017.8784                                             |
| 36                                                    | 52.161                                                |
| <hr style="width: 50px; border: 0.5px solid black;"/> | <hr style="width: 50px; border: 0.5px solid black;"/> |
| 216                                                   | 10178784                                              |
| 108                                                   | 6107270                                               |
| <hr style="width: 50px; border: 0.5px solid black;"/> | 101788                                                |
| 1296                                                  | 20357                                                 |
| .7854                                                 | 5089                                                  |
| <hr style="width: 50px; border: 0.5px solid black;"/> | <hr style="width: 50px; border: 0.5px solid black;"/> |
| 5184                                                  | 1728)164132.88(94 98 Feet.                            |
| 6480                                                  | <hr style="width: 50px; border: 0.5px solid black;"/> |
| 10368                                                 | 8612                                                  |
| 9072                                                  | 17008                                                 |
| <hr style="width: 50px; border: 0.5px solid black;"/> | 14568                                                 |
| 1017.8784 Area Base                                   | <hr style="width: 50px; border: 0.5px solid black;"/> |
|                                                       | 744                                                   |

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the Top-part that is wanting.

.7854

.7854 the Area of Unity.  
 400 the Square of 20.

---

3)314.1600 Area of the lesser Base.

---

104.72 a third Part.  
 268.75 Altitude of the Top-part.

---

52360  
 73304  
 83776  
 62832  
 20944

---

1728)28143.5000(16.28 Feet.

---

10863  
 4955  
 14990  


---

 1166

|                          |       |
|--------------------------|-------|
|                          | Feet. |
| Content of the Whole     | 94.98 |
| Content of the Top-piece | 16.28 |

---

Content of the Frustum 78.7

*Quest. 25.* If the Top-part of a Cone contains 26171 solid Inches, and 200 Inches its Length, and the lower Frustum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base.

|               |              |          |
|---------------|--------------|----------|
| 200           | 159610       | } add.   |
| 200           | 26171        |          |
| <hr/> 40000   | <hr/> 185781 | the Sum. |
| 200           |              |          |
| <hr/> 8000000 |              |          |

26171

Chap. 5. *Practical Questions.* 299

26171 : 8000000 :: 185781 : 56789881, whose  
Cube Root is 384.3 Inches, the Length of the whole  
Cone.

Then find the Diameter of the lesser Base, thus :

$$200)26171$$

$$\begin{array}{r} 130.855 \\ 3 \\ \hline \end{array}$$

392.565 Area of the lesser Base.

Then by Prob. 10. Sect. IX. Chap. 1.

$$1 : 1.2732 :: 392.565$$

$$1.2732$$

$$\begin{array}{r} 785130 \\ \hline \end{array}$$

$$1177695$$

$$2747955$$

$$785130$$

$$392565$$

$$499.8137580(22.35$$

$$4$$

$$42)99$$

$$84$$

$$443)1581$$

$$1329$$

$$4465)25237$$

$$22325$$

$$2912$$

Again,

Lesser Leng. Less. Diam. Gr. Leng. Gr. Diam.

Again, 200 : 22.35 :: 384.3 : 42.94.

|           |                                  |         |
|-----------|----------------------------------|---------|
|           |                                  | Inches. |
| Answer, { | The Length of the whole Cone     | 384.3   |
|           | The Diameter of the greater Base | 43.94   |
|           | The Diameter of the lesser Base  | 22.35   |

*Quest. 26.* There is a Frustum of a Cone, whose solid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears such Proportion to the lesser as 5 to 2; I demand the Diameters.

$$\begin{array}{r} 5 \times 5 = 25 \\ 2 \times 2 = 4 \\ 5 \times 2 = 10 \\ \hline \end{array}$$

The Sum 39

$$\begin{array}{r} 3)12 \\ \hline 4)20(5 \text{ Feet.} \end{array}$$

These 5 Feet are the Triple of a mean Area.

Then, 1 : 1.27324 :: 5 : 6.3662

So the Triple Square of a mean Diameter is 6.3662.

Then, 39 : 6.3662 :: 25 : 4.080897

This 4.080897 is the Square of the greater Diameter, whose Square Root is 2.020123 Feet, which is 24.24147 Inches. Then,

$$5 : 24.24147 :: 2 : 9.69659$$

So the greater Diameter is 24.24147, and the lesser Diameter is 9.69659 Inches.

*Quest. 27.* There is a Room of Wainscot 120 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high (being girt over the Mouldings); there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-boards and Top and Bottom-boards of each Window



Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainscot are contain'd in that Room?

$$\begin{array}{r}
 \text{F. I.} \\
 129 \ 6 \\
 16 \ 9 \\
 \hline
 782 \ 0 \\
 129 \\
 64 \ 9 \\
 32 \ 4 \ 6 \\
 \hline
 2169 \ 1 \ 6
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 7 \ 3 \\
 4 \ 6 \\
 \hline
 29 \ 0 \\
 3 \ 7 \ 6 \\
 \hline
 32 \ 7 \ 6 \\
 16 \ 3 \ 9 \text{ Half.} \\
 \hline
 48 \ 11 \ 3 \\
 2 \\
 \hline
 97 \ 10 \ 6
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 3 \ 3 \\
 7 \ 0 \\
 \hline
 22 \ 9 \\
 11 \ 4 \ 6 \text{ Half.} \\
 \hline
 34 \ 1 \ 6
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 24 \ 6 \\
 1 \ 9 \\
 \hline
 4 \ 6 \\
 18 \ 4 \ 6 \\
 \hline
 42 \ 10 \ 6 \\
 2
 \end{array}$$

$$\begin{array}{r}
 \text{F. I.} \\
 7 \ 3 \\
 5 \ 6 \\
 \hline
 36 \ 3 \\
 3 \ 7 \ 6 \\
 \hline
 39 \ 10 \ 6 \\
 2 \\
 \hline
 79 \ 9 \ 0
 \end{array}$$

$$\begin{array}{r}
 85 \ 9 \bullet \\
 \hline
 \text{F. I.} \\
 3 \ 6 \\
 7 \ 0 \\
 \hline
 24 \ 6 \\
 79 \ 9 \} \text{ add.} \\
 \hline
 104 \ 3
 \end{array}$$

D d

The

|                                |      |    |   |
|--------------------------------|------|----|---|
| The Content of the Room        | 2169 | 1  | 6 |
| The Shutters, at Work and half | 97   | 10 | 6 |
| The Door, at Work and half     | 34   | 1  | 6 |
| The Cheek-boards, &c. °        | 85   | 9  | 0 |

|                                             |      |    |   |
|---------------------------------------------|------|----|---|
| The Sum                                     | 2386 | 10 | 6 |
| The Window-lights and }<br>Door-case deduct | 104  | 3  | 0 |

---

9) 2282    7    6

---

253    5

Answer, 253 Yards 5 Feet.

*Quest. 28.* There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiply'd together, the Product will be 1600, and the Cube of 2 is 8; then say,

$$1600 : 8 :: 18225 : 91.125.$$

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5 the Height; and 8 times 22.5 is 180; the Length.

*Quest. 29.* There is a May-pole, whose Top end was broken off by a Blast of Wind, and the Top end, in falling, struck the Ground at 15 Feet Distance from the Foot of the May-pole, the broken Piece was 39 Feet: Now I demand the Length of the May-pole?

By *Euc. 1.47.* the Square of the Hypotenuse of a right-angled Triangle is equal to the Sum of the Squares of the Base and Perpendicular.

There-

Therefore, from the Square of 39 subtract the Square of 15, the square Root of the Remainder is the Piece standing; to which add the Piece broken off, and you have the whole Length.

$$\begin{array}{r}
 39 \\
 39 \\
 \hline
 351 \\
 117 \\
 \hline
 1521 \\
 225 \\
 \hline
 \dots \\
 1296(36 \\
 9 \\
 \hline
 66)396 \\
 396 \\
 \hline
 \dots
 \end{array}$$

$$\begin{array}{r}
 15 \\
 15 \\
 \hline
 75 \\
 15 \\
 \hline
 225
 \end{array}$$



The Piece standing is 36 Feet.  
 The Piece broken off is 39 Feet.  


---

 The whole Length 75

*Question 50.*

A May-pole there was, whose Height I would know :  
 The Sun shining clear, strait to work I did go :  
 The Length of the Shadow, upon level Ground,  
 Just sixty five Feet, when measur'd, I found ;  
 A Staff I had there, just five Feet in Length ;  
 The Length of its Shadow was four Feet one Tenth :  
 How high was the May-pole, I gladly would know ?  
 And it is the Thing you're desir'd to show.

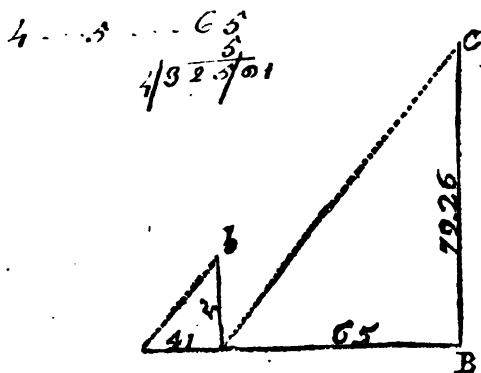
By *Eucl.* 6. 4.

$$a A : A b :: A B : B C.$$

That is,

$$4.1 : 5 :: 65 : 79.26.$$

So I find the Height of the May-pole to be 75 Feet, and a little above 3 Inches.



Here  $A B$  represents the Length of the Shadow of the May-pole, and  $B C$  the May-pole;  $a A$  the Shadow of the Staff, and  $A b$  the Staff.

*Quest.* 31. What will be the Diameter of a Globe, when the Solidity and superficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the superficial Content, we must multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the

## Chap. 5. *Practical Questions.* 305

the Product is the solid Content: therefore, because .5236 is a sixth Part of 3.1416, we must take 6 for the Diameter sought. For if 3.1416 be multiply'd by the Square of 6, *viz.* by 36, the Product will be 113.0976; and if .5236 be multiply'd by the Cube of 6, *viz.* by 216, the Product is likewise 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Answer.

*Quest. 32.* What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Because the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

$$8 : 18 :: 6 : 13.5.$$

So the Diameter sought is  $13\frac{1}{2}$ .

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

$$18 : 8 :: 6 : 2\frac{2}{3}.$$

So then the Diameter will be  $2\frac{2}{3}$ .

The Reason of these Operations, both in this and the last Question, is from Algebra.

*Quest. 33.* There are three Grenado-shells, of such Capacity, that the second Shell will just lie in the Concavity of the first, and the third in the Concavity of the second. The Solidity of the Metal of the first Shell is equal to its Concavity, and the Solidity of the Metal to the second, to the Concavity, is as 7 to 5; and the Solidity of the third, or least Shell's Metal, to its Concavity, is as 9 to 4. Now, supposing the Diameter of the first, or greatest Shell, to be 16

D d 3                      Inches,

Inches, and allowing every solid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two lesser Shells, and the Thickness and Solidity of Metal of every Shell, and also the Weight of every Shell.

The Cube of 16 is 4096; then,

$$1 : .5236 :: 4096 : 2144.6656.$$

The Half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as also the Concavity.

$$.5236 : 1 :: 1072.3328 : 2048.$$

The Cube Root of 2048 is 12.699, which is the Diameter of the second Shell.

The Sum of 7 and 5 is 12; then,

$$12 : 5 :: 1072.3328 : 446.805.$$

This 446.805 is the solid Content of the Concavity of the second.

$$.5236 : 1 :: 446.805 : 853.333.$$

The Cube Root of 853.333 is 9.485, the Diameter of the least Shell.

The Sum of 9 and 4 is 13; then,

$$13 : 4 :: 446.805 : 137.47846.$$

This 137.47846 is the solid Content of the Concavity of the third.

$$.5236 : 1 :: 137.47846 : 262.5639.$$

The Cube Root of 262.5639 is 6.4034, the Diameter of the least Shell's Concavity.

From 16 the Diameter of the greatest.  
Subtr. 12.699 the Diameter of the second.

Rem. 3.301

Half is 1.65 the Thickness of Metal of the greatest.

From

From 12.699 the Diameter of the second,  
Subtr. 9.485 the Diameter of the least.

Rem. 3.214

half is = 1.607 the Thickness of Metal of the second.

From 9.485 the Diameter of the least,  
Subtr. 6.403 the Diameter of the Concavity.

Rem. 3.082

half is = 1.521 the Thickness of Metal of the least.

The Metal of the greatest is 1072.33 solid Inches ;  
which divide by 4, (because every solid Inch is  
a Quarter of a Pound) the Quotient is 268.08  
Pounds.

The Metal of the second is 625.52 solid Inches ;  
which divided by 4, the Quotient is 156.38 Pounds,  
the Weight of the second.

The Metal of the least Shell is 309.32 solid Inches ;  
which divided by 4, the Quotient is 77.33 Pounds,  
the Weight of the least.

The Diam. { second Shell 12.699 } Inches.  
of the { least Shell 9.485 }

The Thickness { greatest 1.65 } Inches.  
of the Metal { second 1.607 }  
of the { least 1.541 }

The Weight { greatest 268.08 } Pounds.  
{ second 156.38 }  
{ least 77.33 }







# A SHORT APPENDIX.



## § I. Of GAUGING.



SHALL not here give the whole Art of Gauging (there being several Books of that Art already in Print, writ by better Hands); but shall only lay down some short practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Vessel upon Oc-  
caſion.



### PROBLEM I.

*To find the ſeveral Multipliers, Diviſors, and Gauge-points belonging to the ſeveral Meaſures now uſed in England.*

282) 1.0000(.003546 Multiplier for Ale Gallons.  
231) 1.0000(.004329 Multiplier for Wine Gallons.  
268.8) 1.000(.0037202 Multiplier for Corn Gallons.  
2150.42) 1.000(.00046502 Multipl. for Corn Baſhels.

So, if the ſolid Inches in any Veſſel be multiply'd by the ſaid Multipliers, the Product will be Gallons  
in

in thr respective Measures; or dividing by the Divisors 282, 231, or 268.8, the Quotient will likewise be Gallons.

*Note,* That 282 solid Inches is a Gallon of Ale or Beer-measure; 231 solid Inches is a Gallon of Wine-measure; 268.8 solid Inches is a Gallon, and 2150.42 solid Inches is a Bushel of Corn-measure.

For circular Area's, the following Multipliers and Divisors are to be used.

282).785398(.002785 Multipliers for Ale Gallons.  
 231).785398(.003399 Multiplier for Wine Gallons.  
 .785398(282.359.05 Divisor for Ale Gallons.  
 .785398(231.294.12 Divisor for Wine Gallons.  
 .785398)2150.42(2738 Divisor for Corn Bushels.

The square Root of the Divisor is the Gauge-point.

|                                                 |   |                  |       |
|-------------------------------------------------|---|------------------|-------|
| The Gauge-point<br>for Squares in               | { | Ale-measure, is  | 16.79 |
|                                                 |   | Wine-measure, is | 15.19 |
|                                                 |   | Malt-bushel, is  | 46 36 |
| The Gauge-point<br>for circular Fi-<br>gures in | { | Ale-measure, is  | 18.95 |
|                                                 |   | Wine-measure, is | 17.15 |
|                                                 |   | Malt-bushel, is  | 52.32 |



## PROBLEM II.

*To find the Area in Ale or Wine Gallons, of any rectilineal plain Figure, whether Triangular, Quadrangular, or Multangular.*

**T**O resolve this Problem, you must, by Chap. I. Part II. find the Area in Inches, and then bring it to Gallons, by dividing that Area in Inches by the proper Divisor, *vis.* by 282 for Ale, or by 231 for Wine; or else by Multiplication, by .003.546 for Ale, or by .004329 for Wine; and the Quotient or Product will be the Area.

*Example*

*Example.* Suppose a Back or Cooler in the Form of a Parallelogram, or long Square, 250 Inches in Length, and 84.5 Inches in Breadth; What is the Area in Ale or Wine Gallons?

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches, which divide by 282, and the Quotient is 74.9 Gallons of Ale; or multiply'd by .003546, the Product is 74.20928 Gallons, nearly the same; and if 21125 be divided by 231, or multiply'd by .004329, it will give 91.44 Gallons of Wine.

### *By Scale and Compasses.*

Extend the Compasses from 282 to 250, that Extent will reach from 84.5 to 74.9: And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

*Note,* The Area's of all Superficies are always to be understood to be 1 Inch deep; otherwise it could not be said, that the Area of such a Parallelogram, Circle, &c. is so many Gallons.

Having found the Area of a Back, or Cooler, the next Thing will be to find out the true Dipping or Gauging-place in that Back, that so the true Quantity of Worts may be computed at any Depth, which may be thus done.

1. When the Bottom of the Back is cover'd all over (of any Depth) with Worts, or other Liquor, then dip it in eight or ten several Places (more or less, according to the Largeness of the Back), as remote and equally distant from each other as you can well do, noting down the wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all those Dips by the Number of Places you dipp'd in, and the Quotient will be the mean Wet of all those Dips.

3. Lastly.

3. Lastly, find out such a Place by the Side of the Back (if you can) that just wets the same with that mean Dip, and make a Notch or Mark there for the true and constant Dipping-place of that Back.

Then if any Quantity of Worts (which covers the whole Back) be deeped or gauged at that Place, and the wet Inches so taken be multiply'd into the Area of the Back in Gallons, the Product will shew how many Gallons of Worts are in that Back at that Time, provided the Sides of the Back do stand at Right-angles with the Bottom.



### PROBLEM III.

*The Diameter of a Circle being given in Inches, to find the Area thereof in Ale or Wine Gallons.*

**I**F the Squaro of the Diameter be multiply'd by .002785 for Ale, or by .003399 for Wine; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, the Products or Quotients will be the respective Ale or Wine Gallons.

*Example.* Suppose the Diameter of a Circle be 32.6 Inches; What will be the Area in Ale or Wine Gallons?

The Square of 32.6 is 1062.76.

Then 359.05)1062.76(2.9599 Area in Ale Gallons.  
And 294.12)1062.76(3.6133 Area in Wine Gall.  
Or  $1062.76 \times .002785 = 2.9598$  Ale Gall.  
And  $1062.76 \times .003399 = 3.6133$  Wine Gall.

By

*By Scale and Compasses.*

Extend the Compasses from 18.95 (the Gauge point for Ale) to 32.6 (the Diameter), that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compasses from 1 to 32.6, that Extent, turn'd twice over from .002785, will at last fall upon 2.9599.

For Wine extend from 17.15, (the Gauge-point for Wine) that Extent, turn'd twice over from 1, will at last fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003399, being turn'd twice over, to 3.6133 Wine Gallons.



## PROBLEM IV.

*The Transverse (or longest Diameter) and the Conjugate (or shortest Diameter) of an Ellipsis (or Oval) being given, to find its Area in Ale or Wine Gallons.*

**I**F the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiply'd by .002785 for Ale, or divided by 294.12, or multiply'd by .003399 for Wine, the Quotient or Product will be the Ale or Wine Gallons requir'd.

*Example.* Suppose the longest Diameter be 81.4 Inches, and the shortest Diameter be 54.6 Inches, What will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale Gallons.  
 294.12)4444.44 15.11 Area in Wine Gallons.  
 Or  $4444.44 \times .002785 = 12.38$  Ale Gallons.  
 And  $4444.44 \times .003399 = 15.11$  Wine Gallons.

### *By Scale and Compasses.*

First, find a mean Proportional between 81.4 and 54.6, by dividing the Distance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval). Then extend the Compasses from 18.95 (the Gauge-point for Ale) to 66.6, that Extent, turn'd twice over from 1, will at last fall upon 12.38, Ale Gallons: And extended from 17.15 (the Gauge-point for Wine) to 66.6, that Extent, turn'd twice over from 1, will reach at last to 15 11 Wine Gallons.



### PROBLEM V.

*To find the Content in Ale or Wine Gallons of any Prism, what Form soever its Base is of.*

**F**IRST, find its solid Content in Inches (by Sect. 1, 2, 3. of Chap. II. Part II.); then divide that Content in Inches by 282 for Ale, or by 231 for Wine; the respective Quotients will be the Content in Wine or Ale Gallons.

Otherwise, you may find the Content of a Prism by finding the Area of its Base in Gallons, (by Problem II. of this Appendix) and multiply that Area by

by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

*Example.* Suppose a Tun, whose Base is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale and Wine Gallons is requir'd.

The Length, Breadth, and Depth, being multiply'd continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons: And by dividing by 2150.4, such a Cistern will be found to hold 35.65 Bushels of Corn.

### *By Scale and Compasses.*

Extend the Compasses from 282 to 36.5, the Breadth of the Base, that Extent will reach from 49.3, its Length, to 6.38 Ale Gallons, the Area of the Base; then extend from 1 to 42.6, the Depth, that Extent will reach from 6.38, the Area of the Base, to 271.8 Gallons, the Content.



## PROBLEM VI.

*To find the Content of a Tun, whose Bases are alike and parallel, but unequal, being the Frustum of a Pyramid.*

**F**IND the Area of each Base, and a mean Proportional between them, and multiply the Sum of those three by one third Part of the Depth or Height, and the Product is the Content.

*Example.* Suppose a Tun, whose Bases are Parallelograms; the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the lesser Base 80, and its Breadth 56; and the Depth of the Tun 42 Inches, the Content in Ale and Wine Gallons is requir'd.

Multiply 100 by 70, the Product is 7000, the Area of the greater Base; and 80 multiply'd by 56, the Product is 4480, the Area of the lesser Base; then multiply the two Areas into each other; and the Product is 31360000, whose square Root is 5600 a geometrical mean Proportional.

|                       |      |        |
|-----------------------|------|--------|
| The greater Area      | 7000 | } add. |
| The lesser Area       | 4480 |        |
| The mean Proportional | 5600 |        |

|                      |             |
|----------------------|-------------|
|                      | 17080       |
| A Third of the Depth | 14          |
|                      | <hr/> 68320 |
|                      | <hr/> 17080 |

282)239120(847.94 A. G.

231)239120(1035.15 W. G.



## PROBLEM VII.

*To find the Content of a Tun, whose Bases are parallel and circular, being the Frustum of a Cone.*

**Y**OU may find the Content as in the last Problem, by multiplying the Sum of the Areas of the two Bases, and a mean Proportional, by one third Part of the Depth.

But it will be a shorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and lesser Diameters add one third Part of the Square of the Difference of the Diameters; that Sum is the Square of a mean Diameter, which, divided by 359.05 for Ale, or by 294.12 for Wine, gives the Area of a mean Circle in Ale or Wine Gallons, which, multiply'd by the Depth, gives the Content.

*Example.* Suppose the greater Diameter 80 Inches, and the lesser Diameter 71 Inches, and the Depth 34 Inches, the Content in Ale or Wine Gallons is requir'd.

Multiply 80 by 71, and the Product is 5680; to which add 27, (a third Part of the Square of the Difference of the Diameters) and the Sum is 5707, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Gallons the Area; which multiply by 34, (the Depth) and the Product will be 540.43 Gallons the Content.

### *By Scale and Compasses.*

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter (though it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great); extend the Compasses from 18.95 (the Gauge-point for Ale) to 75.5, the mean Diameter; that Extent will reach from that 34 (the Depth) to a 4th Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compasses from 17.19 (the Gauge-point for Wine) to 75.5, that Extent will reach from 34, twice turn'd over, to 659.7 Gallons of Wine.

The Method used by the Gaugers for all such Tuns is to take the Diameter in the Middle of every 10 Inches, that is, at five Inches from the Bottom, and at 15. and at 25, &c.

Then they find the Area to every one of these Diameters, and enter them in their Books. Then, when they survey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every ten Inches they take the respective Areas, and remove the separating Point one Place toward the Right Haud; and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and so add all the several Products together, and the Total will be the Gallons of Liquor in the Tun.

*Example.* Suppose the Diameter at 5 Inches from the Bottom be 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area answering to 64 Inches is 11.4078 Gallons; and to 67 Inches, is 12.5023 Gallons; and the Area to 70 Inches, is 13.647 Gallons; and so 73, is 14.8418 Gallons: Then, supposing the Depth of the Liquor in the said Tun be found

found to be 3.6 Inches: Now, to cast up this Gauge, first, in the Area answering to 64 Inches, being multiply'd by 10, that is, by removing the separating Point a Place towards the Right Hand, it will be 114.078 Gallons; and the next will be 125.023; and the next 136.47 Gallons. Now these three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8418 by 3.6, and the Product is 53.4305: Add all these together, and the Sum is the whole Quantity of Liquor in the Tun.

|                                   |         |
|-----------------------------------|---------|
| The Content at 10 Inches deep     | 114.078 |
| The Content at the next 10 Inches | 125.023 |
| The Content at the next 10 Inches | 136.470 |
| The Content of the 3.6 Inches     | 53.430  |

The whole Quantity of Liquor in } 429.001  
the Tun



### PROBLEM VIII.

*To find the Drip or Full of a Tun.*

**S**uppose the Tun last mentioned was so plac'd, that when the Bottom is but just cover'd on one Side, the Liquor is 4 Inches deep on the Side opposite: How much must be allow'd for the Fall of this Tun? That is, How much Liquor is there in the Tun?

The Diameter, in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area answering thereunto is 10.568; which multiply'd by 2 (that is, half 4), the Product is 21.136 Gallons; and so much Liquor will just cover the Bottom.

But

But, suppose it was set so much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the opposite Side just cuts between the Bottoms and Staves; How much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the Top Diameter, and divide the last Product by the Sum of the Diameters, and to the Quotient add the Square of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine; multiply the Quotient by the Depth, the Product is the Content.

The Bottom Diameter of the fore-mentioned Tun is 61 Inches; and the Diameter, at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61 is 3721; which multiply'd by 71.5, the Product is 266051.5; this divided by 132.5, (the Sum of the Diameters) the Quotient is 2007.936; to which add 3721 (the Square of 61), and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiply'd by 30 the Depth, the Product is 159.558 the Gallons of Liquor in the Tun.

When the Frustum of a Cone or Pyramid is cut, by a Diagonal Plain, through the Extremities of the Diameters, as the Liquor in the Tun represents, such Solid is called a Hoof (*Vide Ward's Young Mathematician's Guide, Page 414.*)

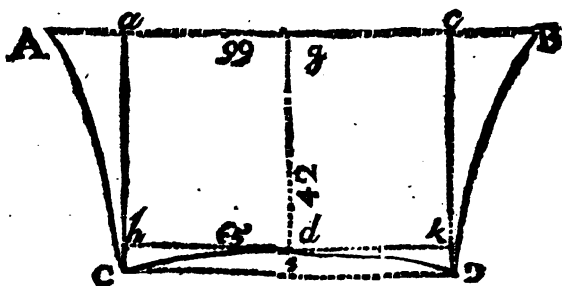
If it be the Hoof of a square Frustum, instead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine. All the rest of the Work is the same.

## PROBLEM IX.

*To Gauge a Copper.*

**L**ET ABCD be a small Copper to be gaug'd.

Take a small Cord or Packthread, make one End fast at A, and extend the other to the opposite Side of the Copper at B, where make it fast, or cause some Person to hold it very strait; then set one End of the Instrument in the Bottom of the Copper at C, and move it to and fro, till you find the nearest Distance to the Thread (as at a): This Distance, a C, is the Depth of the Copper, which suppose to be 47 Inches.



In like manner, set the End of the Rule upon the Top of the Crown at d, and take the nearest Distance to the Thread, as d g, which suppose 42 Inches; this subtracted from a C, 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown.

Measure AB, the Diameter of the Top, which, admit it be 99 Inches; then hold a Thread so as a Plumbet at the End thereof may hang just over C,

C, by which means you will find the Distance A a. Do the like on the other Side; so will you find also the Distance, B; which suppose 17.5 Inches each; add these two together, and subtract their Sum (*viz.* 35) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown. The Diameter which touches the Top of the Crown, may be found by the Sliding-rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards (that is, the Part A B k h, the Depth g d being 42 Inches, you may take the Diameter in the Middle of every 6 Inches of the Depth, which suppose to be as in the second Column of the following Table, the Numbers in the third Column are the respective Areas in Ale Gallons, found by Problem III. the Fourth Column shews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, A B k h; that is, so much as it will hold a'ter the Crown is cover'd.

Now, if the Crown be taken for the Frustum of a Sphere, the Content (by the latter Part of Sect. II. Page 190.) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus:

The Diameter C D, was found to be 64, and the Area to this Diameter is 11.408; this, multiply'd by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Gallons, the Content of the Crown.

The Content of the Part h k D C is 57.935 Gallons; from which subtract the Content of the Crown, 28.52, and the Remainder is 29.415 Gallons, and so much Liquor will just cover the Crown.

| Parts<br>of the<br>Depth | Diameter. | Areas.  | Content<br>of every<br>6 Inches. |
|--------------------------|-----------|---------|----------------------------------|
| 6                        | 95.3      | 25.2945 | 151.767                          |
| 6                        | 90.1      | 22.6095 | 135.657                          |
| 6                        | 85.0      | 20.1223 | 120.734                          |
| 6                        | 80        | 17.8246 | 106.947                          |
| 6                        | 75.2      | 15.7499 | 94.499                           |
| 6                        | 70.5      | 13.8426 | 83.056                           |
| 6                        | 66        | 12.1319 | 72.791                           |
| The Sum                  |           |         | 765.451                          |
| To just cover the Crown  |           |         | 29.415                           |
| The whole Content        |           |         | 794.866                          |

### *By Scale and Compasses.*

You may find the Areas answering to every one of the Diameters, thus :

Extend the Compasses from the Gauge-point to the Diameter ; that Extent, being turn'd twice over from 1, will at last fall upon the Area of that Circle : Or, being turn'd twice over from 6, will give the Content of that 6 Inches of the Depth.

*Example.* Extend the Compasses from 18.95 (the Gauge-point) to 95 3 ; that Extent, turn'd twice over from 6, will at last fall upon 151.76 Gallons, the Content of the first 6 Inches. And so of the rest.

## PROBLEM X.

*To compute the Content of any Close Cask.*

**I**N order to perform this difficult Part of Gauging, the three following Dimensions of the Cask must be truly taken ;

*Fix.*  $\left\{ \begin{array}{l} \text{The Bung-diameter,} \\ \text{The Head-diameter,} \\ \text{The Length of the Cask,} \end{array} \right\} \text{ within the Cask.}$

In taking these Dimensions, it must be carefully observ'd,

1. That the Bung-hole be in the Middle of the Cask ; also, that the Bung-staff, and the Staff opposite to the Bung-hole, are both regular and even within.

2. That the Heads of the Cask are equal, and truly circular ; if so, the Distance between the Inside of the Chine to the Outside of its opposite Staff will be the Head-diameter within the Cask, very near.

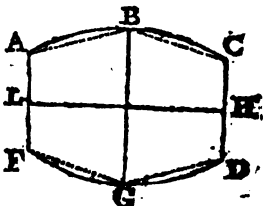
3. With a sliding Pair of Calipers, (made for that Use) take the shortest Distance, or Length, between the Outfides of the two Heads ; from that Length subtract  $1 \frac{1}{4}$  Inch (more or less, according to the Largeness of the Cask) for the Thickness of the Head : The Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length, by putting a strait Rod in at the Tap-hole, and allow for the Thickness of the Head.

Now by these Dimensions, one would think the Content of the Cask was perfectly limited ; but it will be easy to perceive, by the following Figure,



that the Diameters and Length of one Cask may be equal to those of another, and yet one of those Casks may contain several Gallons more than the other.

As for Instance, the Figure ABCDF is suppos'd to represent a Cask: Then it is plain, that if the outward curve Lines, ABC, and FGD, are the Bounds or Staves of the Cask, it must needs hold more than if the inner prick'd Lines were the Bounds or Staves; and yet the Bung-diameter BG, and Head-diameters CD and AF, and Length LH, are the same in both those Casks.



Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Casks can be gauged: And therefore Gaugers do usually suppose every Cask to be in some of these Forms:

1. The middle Frustum of a Spheroid.
2. The middle Frustum of a parabolic Spindle.
3. The lower Frustums of two equal parabolic Conoids.
4. The lower Frustums of two equal Cones.

1. If the Staves of the Cask be very much curved (as the outward Lines of the last Figure), then the Cask is supposed to be the middle Frustum of a Spheroid.

2. If the Staves (between the Bung and Head) be something less curved, then the Cask is taken to be the middle Frustum of a parabolic Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Frustums of two equal parabolic Conoids, abutting

F f

ting

ting, or joining together, upon one common Base.

4. If the Staves (between the Bung and Head) be strait (as the prick'd Lines in the last Figure), then the Cask is taken to be the lower Frustrums of two equal Cones, abutting or joining together upon one common Base.

There are several Rules laid down in Books of Gauging, for finding the Content of each several Form; but I think the shortest and most practical Way is, to find such a mean Diameter, which will reduce the proposed Cask to a Cylinder. Thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the second Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-diameter, and the Sum is a mean Diameter.

*Example.* Suppose the Bung-diameter be 32 Inches, the Head-diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is requir'd.

The Difference between the Bung and Head-diameter is 8; which multiply'd by .7, the Product is 5.6; which added to the Head-diameter, the Sum is 29.6, the mean Diameter: The Area answering thereunto will be found (by Prob. III.) to be 2.44 Ale Gallons; which multiply'd by the Length, the Product is 97.4 Gallons: and so much is the Content, if it be the first Form.

Again, if the Difference of the Diameters 8 be multiply'd by .65, the Product will be 5.2; which added to the Head-diameter, the Sum is 29.2, for the mean Diameter; and the Area answering thereunto is 2.3746 Gallons; which multiply'd by 40 (the Length), the Product is 94.98 Gallons, the Content, if it be of the second Form.

Again, if the Difference 8 be multiplied by .6, the Product is 4.8; which added to the Head-diameter, the Sum is 28.8, the mean Diameter; the Area thereunto is 2.31 Gallons; which, multiply'd by 40, gives the Content 92.4 Gallons, for the third Form.

Again, the Difference 8, multiply'd by .55, the Product is 4.4; which added to the Head-diameter, makes the mean Diameter 28.4; the Area thereof is 2.2463; which multiply'd by 40, the Product is 89.85 Gallons, for the fourth Form.

### *By Scale and Compasses.*

Extend the Compasses from the Gauge-point 18.95, to the first mean Diameter 29.6; that Extent will reach from the Length 40, to a fourth Number, and then to the Content, 97.4 Gallons.

Again, Extend from 28.95 to 29.2 (the second mean Diameter), that Extent, turn'd twice over from 40, will at last fall upon 94.98 Gallons.

Again, Extend from 18.95 to 28.8 (the third mean Diameter) that Extent, turn'd twice over from 40, will at last fall upon 92.4 Gallons.

Again, Extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turn'd twice over from 40, will at last fall upon 89.85 Gallons.

Altho' I have all along made use of the Line of Numbers upon the common Two Foot, or Eighteen-Inch Rules, for the Reason mentioned in the *Preface*; yet the Rules may easily be applied to the Sliding-rule, thus: To find the Area of a Circle in Gallons, set the Gauge-point upon D (that is, a single Line of Numbers, to 1 upon C (that is, a double Line);

F f 2

then

then against any Diameter upon  $D_1$  is the Area upon C, thus :

To find the Content of the Cask, last-mention'd, the first Form.

Set the Gauge-point 18.95 upon D, to the Length 40 upon C ; then (against the mean Diameter) 29.6 upon D, is 97.4 Gallons, the Content upon C.

And against 29.2 (the next mean Diameter on D, is 94.98 Gallons on C,

And against 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And against 28.4 (the last mean Diameter) on D, is 89.85 Gallons on C.

*All done without removing the Slider.*

A TABLE

*A TABLE of the Segment of a Circle,  
whose Area is Unity.*

| V. S. Segm | V. S. Se m | V. S. Segm | V. S. Segm |
|------------|------------|------------|------------|
| 1 .0017    | 99 .9983   | 26 .2056   | 74 .7924   |
| 2 .0048    | 98 .9952   | 27 .2178   | 73 .7832   |
| 3 .0087    | 97 .9913   | 28 .2292   | 72 .7708   |
| 4 .0134    | 96 .9866   | 29 .2407   | 71 .7593   |
| 5 .0187    | 95 .9813   | 30 .2523   | 70 .7477   |
| 6 .0245    | 94 .9755   | 31 .2640   | 69 .7360   |
| 7 .0308    | 93 .9692   | 32 .2759   | 68 .7241   |
| 8 .0375    | 92 .9625   | 33 .2878   | 67 .7122   |
| 9 .0446    | 91 .9554   | 34 .2998   | 66 .7002   |
| 10 .0520   | 90 .9480   | 35 .3119   | 65 .6881   |
| 11 .0598   | 89 .9402   | 36 .3241   | 64 .6759   |
| 12 .0680   | 88 .9320   | 37 .3364   | 63 .6630   |
| 13 .0764   | 87 .9236   | 38 .3487   | 62 .6513   |
| 14 .0851   | 86 .9149   | 39 .3611   | 61 .6389   |
| 15 .0941   | 85 .9059   | 40 .3735   | 60 .6265   |
| 16 .1033   | 84 .8967   | 41 .3860   | 59 .6140   |
| 17 .1127   | 83 .8873   | 42 .3986   | 58 .6014   |
| 18 .1224   | 82 .8776   | 43 .4112   | 57 .5888   |
| 19 .1323   | 81 .8677   | 44 .4238   | 56 .5762   |
| 20 .1424   | 80 .8576   | 45 .4364   | 55 .5636   |
| 21 .1526   | 79 .8474   | 46 .4491   | 54 .5509   |
| 22 .1631   | 78 .8369   | 47 .4618   | 53 .5382   |
| 23 .1737   | 77 .8263   | 48 .4745   | 52 .5255   |
| 24 .1845   | 76 .8155   | 49 .4873   | 51 .5127   |
| 25 .1955   | 75 .8045   | 50 .5000   | 50 .5000   |

### *The Use of the Table of Segments.*

Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whose Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.

#### *The RULE is;*

To the wet or dry Inches of the Bung-diameter, add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiply'd by the whole Content of the Cask, the Product shews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Cask in Form of a Cylinder, whose Bung-diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Cask?

Divide the dry Inches, 13 by 29, the Bung diameter, and the Quotient is .448; find the two first Figures .44 under V. S. and the Segment again<sup>st</sup> it is .4238; to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiply'd by the Content of the Cask, the Product will be 34.664 Gallons; and so much the Cask wants of being full.

*Note,* If the Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a spheroidal Cask, then use the following Method.

1. By

1. By the Bung and Head-diameter, find such a mean Diameter as, you judge, will reduce the propos'd Cask to a Cylinder, and then find its Content.

2. From the Bung-diameter subtract the mean Diameter, and take half the Difference.

3. From the wet Inches subtract the said Half-difference; reserve this Difference, then use this Proportion :

As the mean Diameter is to 100  
(the Diameter of the tabular Circle),  
so is the reserv'd Difference,  
to a versed Sine in the Table.

Then, if the tabular Segment be multiply'd into the Content (as before) the Product will be the Quantity of Liquor in the Cask.

*Example.* Let the Cask be the same as in Page 325, of the first Form, where the Bung-diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and suppose the wet Inches 19, to find the Quantity of Liquor in the Cask.

From 32  
subtr. 29.6

Rem. 2.4

Half 1.2

From 19  
subtr. 1.2

Rem. 17.8 reserv'd.

29.6 : 100 :: 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiply'd by 97.4 the Content, the Product is 61 Gallons, the Quantity of Liquor in the Cask.

*Example.* There is a Cistern, whose Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Bushels, the Area of the Bottom at 1 Inch deep; which multiply'd by the Depth 43.6, the Product is 91.98 Bushels, the Content.

*Example.* Suppose a Quantity of Malt upon a Floor, whose Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; How many Bushels are there?

Multiply 245 by 184, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Base; which multiply'd by the mean Depth, the Product 117.4 Bushels, the Content.

### *By the Sliding-rule.*

There is an inverted Line of Numbers upon some Sliding-rules, mark'd with the Letter M, which was contriv'd purposely for Gauging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of these are of equal Radius, and all work together at once: Thus set the Length and Breadth against one another upon the inverted Line, and that which slides by it; then, on the other Edge of the Rule against the Depth, you will find the Content in Bushels. Thus, in the first Example, set 54 upon the Slider against 84, upon the inverted Line; and then, against 43.6 upon the other Part of the Rule, is 91.98 upon the Slider.

Again, in the second Example, set 184 upon the Slider to 245 upon the inverted Line; and against 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.





## § II. Of LAND-MEASURING.



SHALL not here give the whole Art of *Surveying*, but such practical Rules only as may be useful to the Country Grasers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only (and for want of that, with a Pole or Stick, of half a Rod in Length).



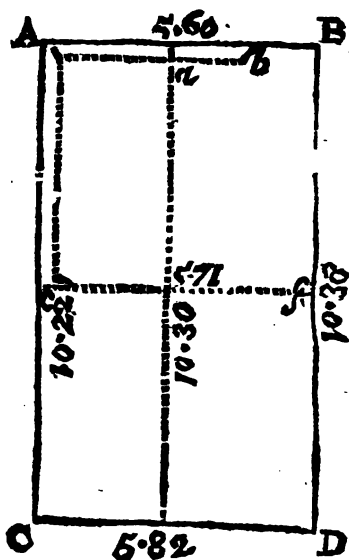
## PROBLEM I.

*To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or long Square, or what is something near that Form.*

TO know whether any Angle in a Field be a Right-angle, or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or square; and, with a Saw, cut two Kerfs, crossing each other at Right-angles; and bore a Hole in the Middle of the Back-side, to put it upon the End of a Stick. This will represent the Instrument call'd a Cross.

Suppose

Suppose you would observe the Angle A, to know whether it be a Right-angle (or near thereunto); prick up your Stick, with the Cross upon it, a little Distance from the Fence, as at *a*; and having set up two Marks, as at *b* and *c*, of equal Distance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.



To measure such a Piece of Ground as this Figure above: If you measure round, and add the opposite Sides together, and take half the Sum, (if they be not equal); or else measure down about the Middle of the Length, and Middle of the Breadth; thus, the Side AB being measured, it will be 5.60 (that is, 5 Chains and 60 Links); and the opposite Side CD is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38; and the Side AC 10.22; and the half Sum thereof is 10.30 (it will be the same thing, if you measure about the Middle of the Length, and Middle of the Breadth); then multiply this mean Length and mean Breadth together, *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10 (because 10 square Chains is an Acre) by removing the separating Point one Place towards the Left hand

## Sect. II. Of Land-Measuring.

337

Left-hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 15 Places, and it will be 3.52520; which 3 towards the Left-hand are 3 Rods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left-hand are 21 Perches.

A. R. P.

So the whole Content is ——— 5 3 21

See the Work.

|          |          |
|----------|----------|
| 5.71     |          |
| 10.30    |          |
| -----    |          |
| 17130    |          |
| 571      | A. R. P. |
| -----    | 5 3 21   |
| 5.88130  |          |
| 4        |          |
| -----    |          |
| 3.52520  |          |
| 40       |          |
| -----    |          |
| 21.00800 |          |
| -----    |          |

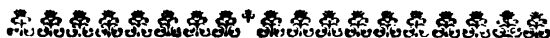
*Note,* The Chain here made use of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But because every Man that may have Occasion to measure a Piece of Land, can't procure a Chain, I will therefore shew how you may measure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; Which Stick divide into five equal Parts, so will the whole Rod be divided into ten Parts, and will be thereby adapted to Decimal Arithmetick.

G g

But

But because each of those Parts of the Stick are something large, (each Part being 19 Inches and 8 Tenths) it will be necessary to take your Dimensions to half of one of those Parts; and then, for that half Part. set 5 in the Place of Seconds, thus; suppose 3 Parts and a half, set it down thus .35.



## PROBLEM II.

**L**ET us suppose a Field in the Form of a long Square, whose Length is 45 Rods 5 Parts and a Half, and the Breadth 31 Rods 4 Parts and a Half; What is the Content?

Multiply the Length and Breadth together, and divide the Product by 160, (because 160 square Rods is an Acre) and the Quotient is Acres.

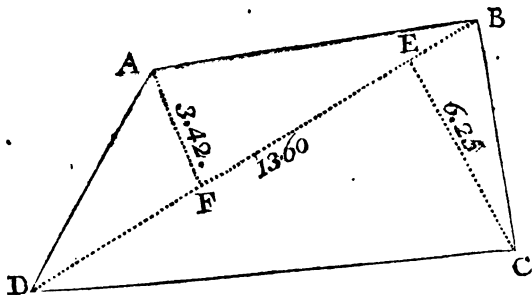
$$\begin{array}{r}
 45.55 \\
 31.45 \\
 \hline
 22775 \\
 18220 \\
 4555 \\
 13665 \\
 \hline
 1432.5475
 \end{array}$$

$$\begin{array}{r}
 16|0)14312(8 \quad \text{A. R. P.} \\
 \underline{128} \quad \text{Facit } 8 \quad 3 \quad 32 \\
 4|0)1512(3 \\
 \underline{12} \\
 32
 \end{array}$$



### PROBLEM III.

**S**uppose a Piece of Ground in the Form of a Trapezium; the Diagonal BD 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links; what is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars. See Sect. VI. of Chap. I. Part II.

340

CE=6.25

AF=3.42

Sum 9.67

Half 4.83

Appendix.

13.60=BD

4.83

4080

10880

5440

6.56880

4

2.227520

40

11.00800

A. R. P.

Facit 6 2 11

By Rods, thus ;

CE=15 Rods.

AF=13.68

Sum 38.68

Half 19.34

19.34

544=BD

7736

7736

9670

16|105|2.096(6

96

4|09|2(2

8

12

A. R. P.

Facit 6 2 12

*To take the Dimensions of the Field.*

Begin at the Angle B, and measure in a direct Line towards D; but when you come at E, set up your Cross, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then are you just in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards, and forwards till it does so; then measure the Perpendicular, and set down the Chains and Links, or the Rods and Parts; then continue your Measure towards D; but when you come to F, set up your Cross, and try (as is above directed), whether you be in the Place where the Perpendicular will fall. Then measure the Perpendicular A F, and set down the Chain and Links or Rods and Parts; then continue your Measure to D, and set down the Measure of the whole Diagonal. This Way of Measuring is very exact and true; but the common Way used by the Grasers and Farmers, is to measure round the Field, and to take half the Sum of the opposite Sides for a mean Side; but the last-mentioned Piece of Ground, being measur'd so, will come to

A. R. P.

R. P.

7 0 22, which is 2 10 more than the Truth.

## PROBLEM I.

*How to measure an Irregular Field.*

**T**HE Way to measure irregular Land, is to divide it into Trapeziums and Triangles, thus :

First, view over the Field, and set up Marks at every Angle, and by those Marks you may see where to have a Trapezium, as A B C I in the following Figure.

Then





the Perpendicular will fall; then measure the Per-  
Ch. L.

pendicular b B, which is 2.06; then continue your Measure to C, and you will find the whole Diagonal  
Ch. L.

9.42.

Then proceed to measure the Trapezium CDHI, beginning at C, and measuring along the Diagonal Line towards A; but when you come at (d), set up your Cross, and try if you be in the Place where the Perpendicular will fall: Measure the Perpendicular d D,

Ch. L.

which is 1.46, and then measure forward till you come at (c), and there, with your Cross, try if you be right in the Place where the Perpendicular will fall, and measure the Perpendicular c I, which is 3 Chains; and from (c) continue your Measure to H,

Ch. L.

and you will find the whole Diagonal 12.36.

Then proceed to measure the Trapezium HGED, beginning at H, and measuring along the Diagonal Line towards E; but when you come to (f), try with your Cross if you be in the Place where the Perpendicular will fall; and measure the Perpendicular f G, which is 4.48; then continue on your Measure from (f) till you come to (g), and there try if you be in a Square with the Perpendicular g D; and

Ch. L.

measure the said Perpendicular, which is 2.94; then measure on from (g) to E, and you will find the

Ch. L.

whole Diagonal to be 11.34.

Then measure the Triangle EFG, beginning at E, and measuring along the Base EG till you come at (h), and there with your Cross try if you be in the Place where the Perpendicular will fall; and mea-

Ch. L.

sure the Perpendicular h F, which is 3.14, continue your Measure to G, and you will find the whole Base

Ch. L.

to be 9.12; so you have finish'd your whole Field.

I have

## Se&. II. *Of Land-Measuring.* 345

I have been the larger upon the Explanation of this Problem, because most Grounds lie in such irregular Forms.

Cast up the three Trapeziums severally, and also the Triangle; and add all the several Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

|           |                              |                                   |
|-----------|------------------------------|-----------------------------------|
| b B=2.06  | 9.42                         | See Se&. VI. Chap. I.<br>Part II. |
| a I=4.82  | 3.44                         |                                   |
| Sum 6.88  | 3768                         |                                   |
|           | 3768                         |                                   |
| half 3.44 | 2826                         |                                   |
|           | <u>3.24048=Area of ABCI.</u> |                                   |

|           |                              |
|-----------|------------------------------|
| d D=1.46  | 12.36                        |
| c I=3.00  | 2.23                         |
| Sum 4.46  | 3708                         |
|           | 2472                         |
| half 2.23 | 2472                         |
|           | <u>2.75628=Area of CIHD.</u> |

|           |                              |
|-----------|------------------------------|
| f G=4.48  | 11.34                        |
| g D=2.94  | 3.71                         |
| Sum 7.42  | 1134                         |
|           | 7938                         |
| half 3.71 | 3402                         |
|           | <u>4.20714=Area of HGED.</u> |

Base

Base = 9.12

Half = 4.56

Perpend. 3.4

1824

456

1368

See Sect. V. Chap. I.  
Part II.

1.43184 = Area of the Triangle EFG.

3.24048 = Area of ABCI.

2.75628 = Area of CIHD.

4.20714 = Area of HGED.

Sum 11.63574 = Area of the Whole.

4

2.54296

40

21.71840

A. R. P.

Facit 11 2 21

## F I N I S.

Eccle. 1. 1. 2. A perfect man.

The first Adam was a perfect before  
the fall. Gen. 1: 26, 27. and Jesus  
Christ the second Adam was a  
perfect man. Heb. 9: 14. Said  
Jesus the prince of this world com-  
eth, and hath nothing in me,  
John 14: 30. And when I am truth-  
fully say the same I will be fit  
to have it. Rom. 8: 9. 1 Cor. 15: 45-49.